Automatic threshold adjustment for limit cycles holding a specified stability in hybrid systems

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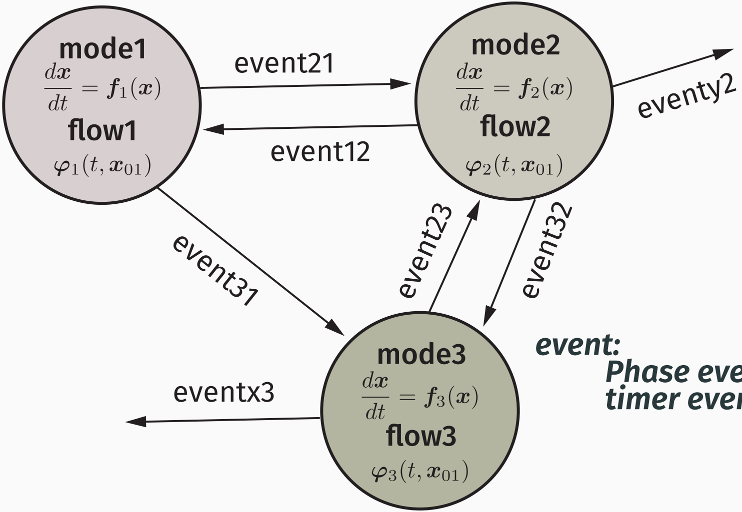
Hybrid systems

Hybrid systems—Finite automata



event:
Phase event
timer event

Our original contributions

Computation of bifurcation sets in hybrid systems

- T. Kousaka, T. Ueta, and H. Kawakami, “ Bifurcation of switched nonlinear dynamical systems,” IEEE Trans. CAS-II, **46**, no. 7, pp. 878–885, 1999.
- ...
- Y. Miino, D. Ito, and T. Ueta, “A computation method for non-autonomous systems with discontinuous characteristics,” Chaos, Solitons & Fractals, **77**, 8, pp. 277–285, 2015.

Problem description

Given the hybrid system:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x})$$

where, $\boldsymbol{f}$ contains several different dynamical systems aligned by a threshold $\boldsymbol{x}_{\mathrm{th}} \in \boldsymbol{R}$

Assume that a solution is written by:

$$\boldsymbol{x}(t) = \varphi(t, \boldsymbol{x}_0), \quad \boldsymbol{x}(0) = \varphi(0, \boldsymbol{x}_0) = \boldsymbol{x}_0.$$

Our speciality

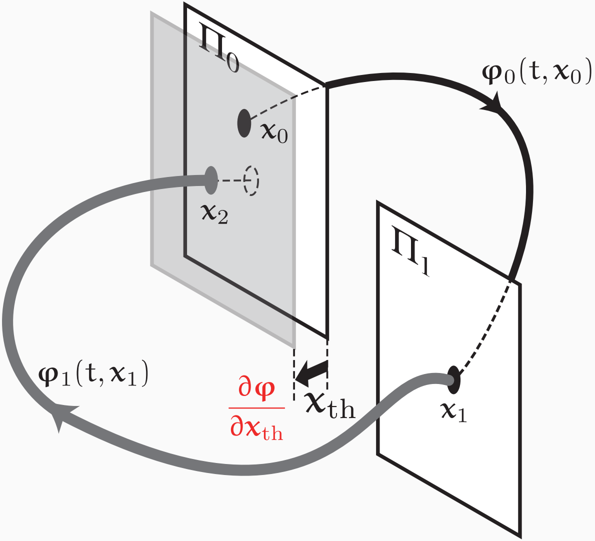
$$\frac{d\boldsymbol{x}}{dt} = \begin{cases} \boldsymbol{f}_0(\boldsymbol{x}) & \text{if} \quad \boldsymbol{x} \in R_1 & \Leftrightarrow \varphi_0 \\ \boldsymbol{f}_1(\boldsymbol{x}) & \text{if} \quad \boldsymbol{x} \in R_2 & \Leftrightarrow \varphi_1 \\ \vdots & & \\ \boldsymbol{f}_{m-1}(\boldsymbol{x}) & \text{if} \quad \boldsymbol{x} \in R_{m-1} & \Leftrightarrow \varphi_{m-1} \end{cases}$$

where R_i is a region of i.

We can evaluate $\frac{\partial \varphi_j}{\partial \boldsymbol{x}_{\mathrm{th}j}}$ even though $\boldsymbol{x}_{\mathrm{th}j}$ are implicit parameters!

D. Ito, T. Ueta, and K. Aihara: IEICE Trans. Fundum. JA-94, No. 8, 2011. (Japanese)

Perturbations



Poincaré mapping and its fixed point

The fixed point of the Poincaré map: $T(\boldsymbol{x})$:

$$T(\boldsymbol{x}_0) - \boldsymbol{x}_0 = \mathbf{0} \tag{1}$$

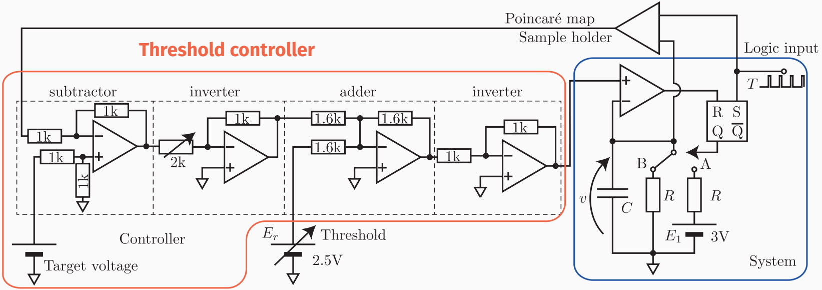
The characteristic equation:

$$\chi(\mu) = \det(DT(\boldsymbol{x}_0) - \mu I) = 0$$

Controlling chaos for hybrid systems

Controlling chaos by a threshold

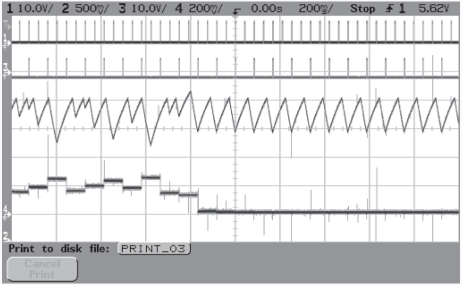
D. Ito, et al. Int. J. Bifur. Chaos, **24**, 10, 2014.



Controlling chaos by a threshold

D. Ito, et al. Int. J. Bifur. Chaos, **24**, 10, 2014.

- Controller perturbes the threshold value
- The threshold value converges when completed



← clock

← Poincaré map

← $v(t)$

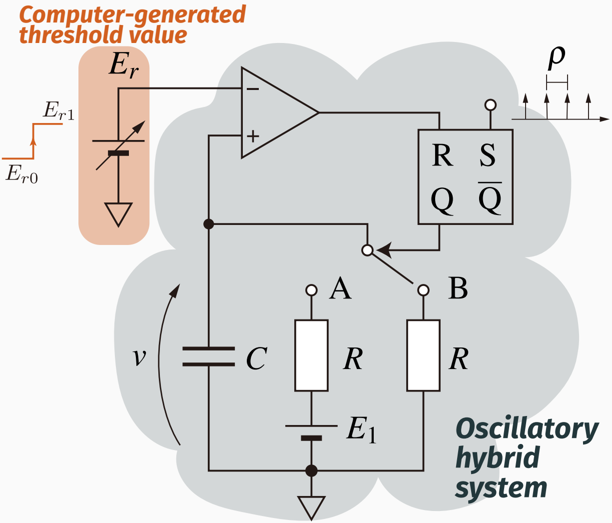
← Threshold value E_r

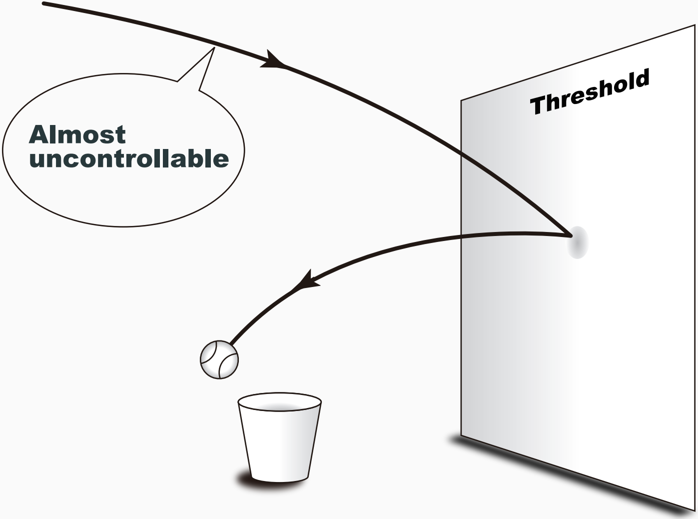
In this talk, we propose...

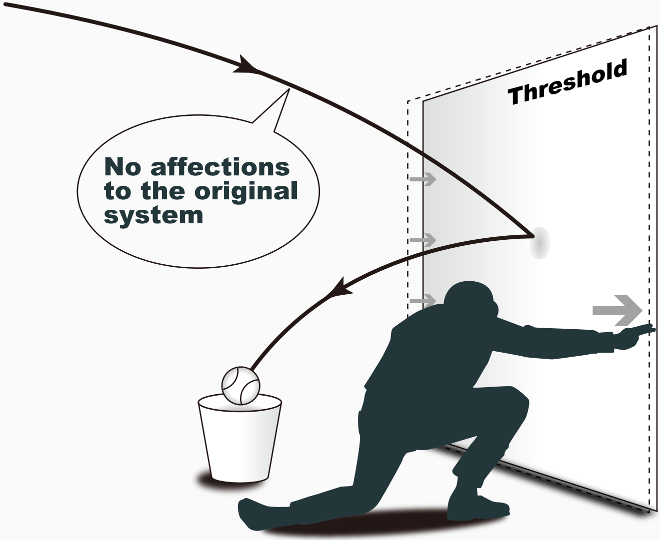
Stability design

Solve a threshold value of the limit cycle whose multiplier satisfies the specific stability.

Open-loop control; an example




Almost
uncontrollable
Threshold



No affections to the original system
Threshold

Example: van der Pol oscillator with a threshold

Switched van der Pol oscillator:

$$\frac{d\boldsymbol{x}}{dt} = \begin{cases} \boldsymbol{f}_0(\boldsymbol{x}, \epsilon, \omega) & \text{if} \quad q(\boldsymbol{x}) \leqslant 0 \\ \boldsymbol{f}_1(\boldsymbol{x}, \epsilon, \omega) & \text{otherwise,} \end{cases} \tag{2}$$

where,

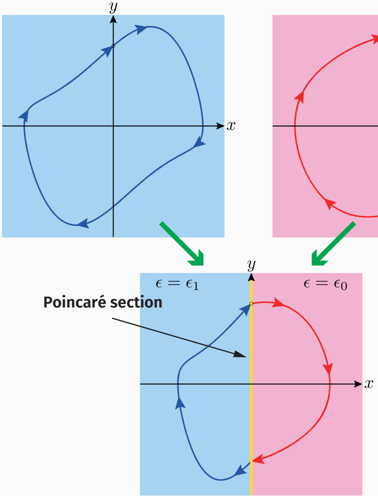
$$\boldsymbol{f}_0 = \begin{pmatrix} y \\ \epsilon_0(1 - x^2)y - \omega x \end{pmatrix}, \quad \boldsymbol{f}_1 = \begin{pmatrix} y \\ \epsilon_1(1 - x^2)y - \omega x \end{pmatrix}.$$

$x_{\mathrm{th}} \in \boldsymbol{R}$ is a threshold value, $q(\boldsymbol{x}) = x - x_{\mathrm{th}} \in \boldsymbol{R}$.

A solution:

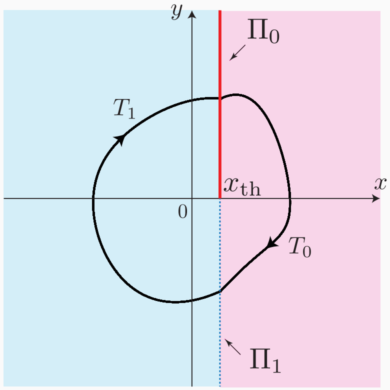
$$\boldsymbol{x}_i(t) = \varphi_i(t, \boldsymbol{x}_{i0}), \quad \boldsymbol{x}_i(0) = \boldsymbol{x}_{i0}. \tag{3}$$

Limit cycles of hybrid dynamical systems



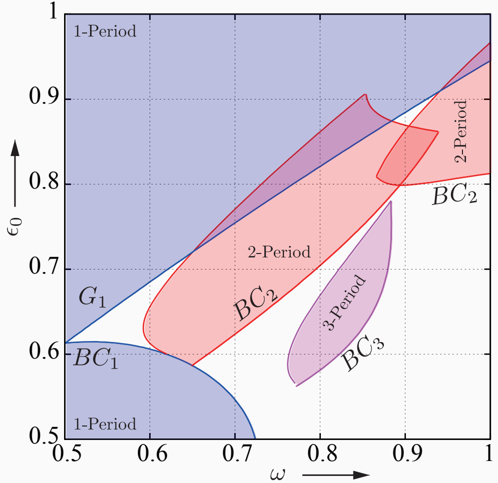
The threshold value may change the stability of the cycle.

Sample phase portrait



$\epsilon_0 = 0.2$, $\epsilon_1 = 0.1$, and $x_{\text{th}} = 0.1$.

Bifurcation diagram



Derivative of T

The derivative with the initial value of the Poincaré map:

$$\frac{\partial T}{\partial \boldsymbol{x}_0} = \prod_{i=0}^{m} \left. \frac{\partial T_{1-i}}{\partial \boldsymbol{x}_{1-i}} \right|_{t=\tau_{1-i}} . \tag{4}$$

Each Jacobian matrix is given by:

$$\frac{\partial T_i}{\partial \boldsymbol{x}_i} = \left[I_n - \frac{1}{\frac{\partial q}{\partial \boldsymbol{x}} \cdot \boldsymbol{f}_i} \boldsymbol{f}_i \frac{\partial q}{\partial \boldsymbol{x}} \right] \frac{\partial \varphi_i}{\partial \boldsymbol{x}_i} \tag{5}$$

hoge

$$\begin{aligned} \Pi_0 &= \left\{ (x, y) \mid q(x) = 0, \frac{dx}{dt} > 0 \right\}, \\ \Pi_1 &= \left\{ (x, y) \mid q(x) = 0, \frac{dx}{dt} < 0 \right\}. \end{aligned} \tag{6}$$

The local map is written by

$$\begin{aligned} &T_0 : \Pi_0 \to \Pi_1; \quad \boldsymbol{x}_0 \mapsto \boldsymbol{x}_1 = \varphi_0(\tau_0, \boldsymbol{x}_0), \\ &T_1 : \Pi_1 \to \Pi_0; \quad \boldsymbol{x}_1 \mapsto \varphi_1(\tau_1, \boldsymbol{x}_1). \end{aligned} \tag{7}$$

The Poincaré map:

$$T(\boldsymbol{x}_0) = T_1 \circ T_0(\boldsymbol{x}_0). \tag{8}$$

Local coodinate

The local coordinate: $u \in \Sigma_0 \subset \boldsymbol{R}$

A projection p and an embedding map p^{-1}:

$$p^{-1} : \Sigma_0 \to \Pi_0, \; p : \Pi_0 \to \Sigma_0. \tag{9}$$

The Poincaré mapping on the local coordinate:

$$T_\ell : \Sigma_0 \to \Sigma_0; \quad u \mapsto p \circ T \circ p^{-1}(u). \tag{10}$$

The fixed point of the Poincaré mapping u^*

$$T_\ell(u_0) - u_0 = 0. \tag{11}$$

Jacobian matrix

The Jacobian matrix is given by

$$\frac{\partial T_\ell}{\partial u_0} = DT_\ell(u_0) = \frac{\partial p}{\partial \boldsymbol{x}} \frac{\partial T}{\partial \boldsymbol{x}_0} \frac{\partial p^{-1}}{\partial u}. \tag{12}$$

The characteristic equation for the fixed point is given by

$$\chi_\ell(\mu) = \det\left(DT_\ell - \mu^* I\right) = 0, \tag{13}$$

where μ^* is specified multiplier. We can obtain a parameter x_{th} with the specified μ^* by solving Eq. (11) and (13).

Computation of the fixed point

The derivative with x_{th} is required in Newton's method and is obtained from the previous study as follows:

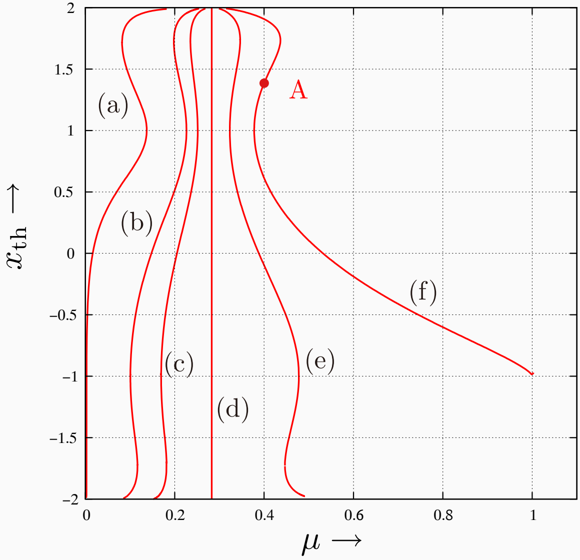
$$\frac{\partial T_\ell}{\partial x_{\mathrm{th}}} = \frac{\partial p}{\partial \boldsymbol{x}} \frac{\partial T}{\partial x_{\mathrm{th}}}. \tag{14}$$

Now we are ready to solve them for $\boldsymbol{u}_0$ and x_{th}.

Control procedure

1. Give the desired multiplier
2. Compute the threshold value satisfying the multiplier
3. Change the threshold value

Isocline, $\epsilon_0 = 0.2$



(a) $\epsilon_1 = 1.0$

(b) $\epsilon_1 = 0.4$

(c) $\epsilon_1 = 0.3$

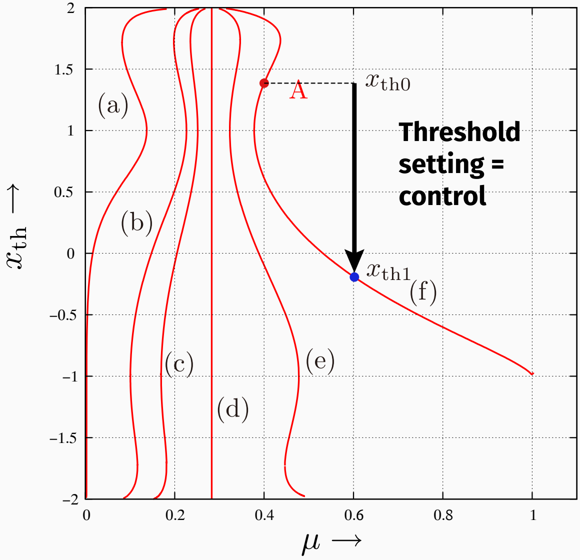
(d) $\epsilon_1 = 0.2$

(e) $\epsilon_1 = 0.1$

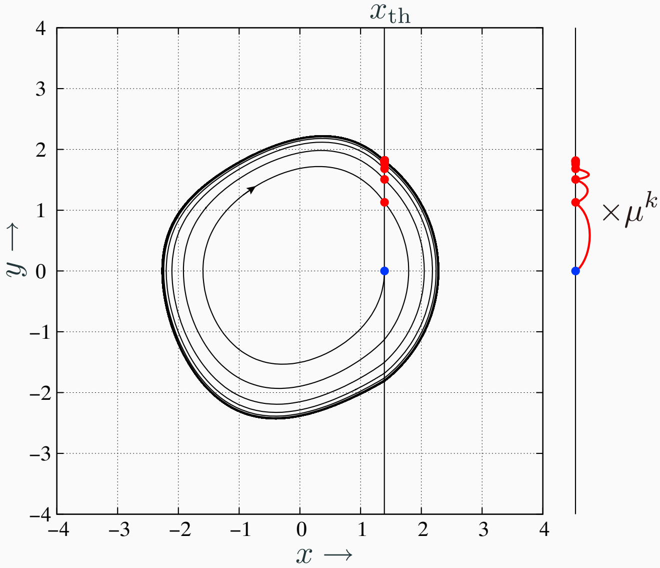
(f) $\epsilon_1 = 0.0$

$E_r \Leftrightarrow x_{\text{th}}$

Isocline, $\epsilon_0 = 0.2$



At point A, $\mu = 0.4$



Features and advantages

- Change of the threshold value: an implicit parameter
  - No control energy is consumed
  - No explicit parameter is changed — system unchanged
  - Transition: almost original system behavior
- Non-dynamical controller is realized

Concluding Remarks

- Stability with the threshold value for the given hybrid systems are computable
- A limit cycle holding a desirable stability is controlled by the threshold value
- The control scheme is easy: just change a threshold value stepwisely
- **NO TRANSITION**, No affection to the state variables, basically.

References

- K. Aihara, J. Imura and T. Ueta (eds), *Analysis and Control of Complex Dynamical Systems*, Springer, Tokyo, Mar. 2015.
- D. Ito, T. Ueta, T. Kousaka, J. Imura, and K. Aihara, “Controlling Chaos of Hybrid Systems by Variable Threshold Values,” Int. J. Bifurcation and Chaos, Vol. 24, No. 10, 1450125, 2014.
- T. Ueta, S. Tsuji, T. Yoshinaga, and H. Kawakami, “Calculation of the isocline for the fixed point with a specified argument of complex multipliers,” in Proc. IEEE/ISCAS 2001, Vol. 2, pp. 755–758, 2001.

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科研費 KAKENHI

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