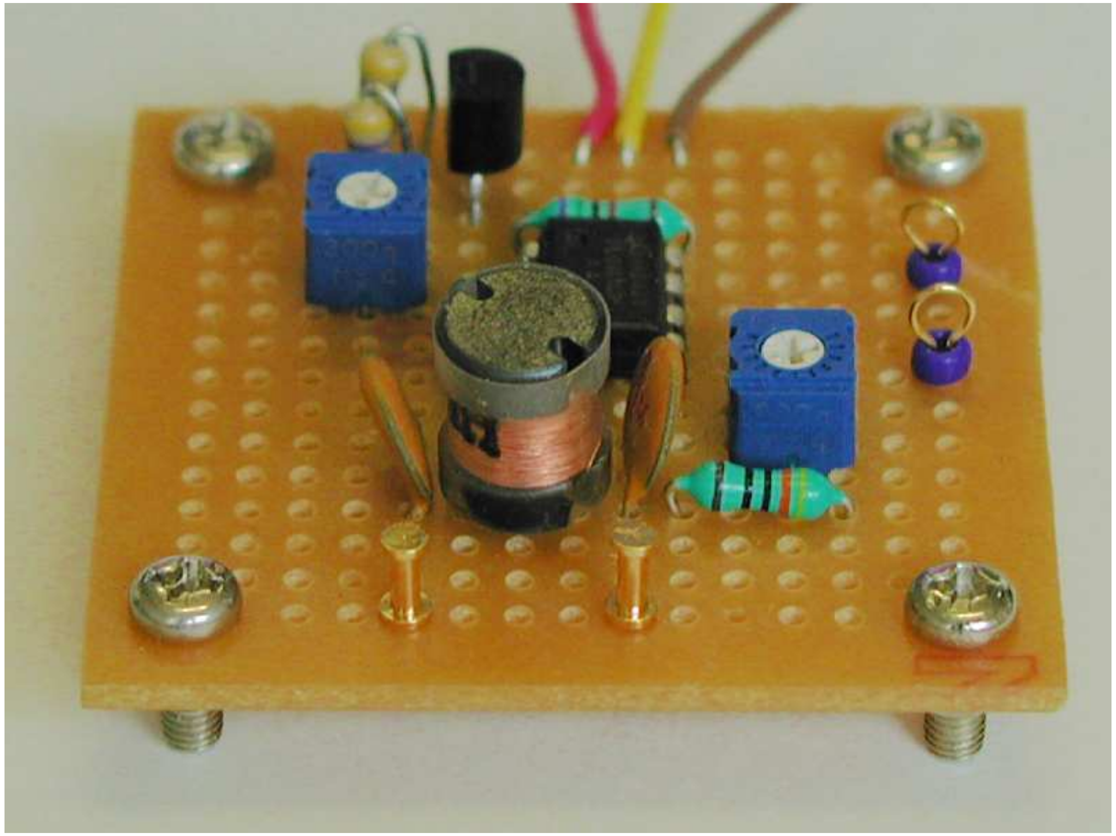
Bifurcation and Chaos in the Extended BVP Oscillator



T. Ueta and H. Kawakami

Tokushima University, Japan

Coupled oscillators

- practical industrial applications models
- biological activities

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Decomposite a complex nonlinear dynamics into unit oscillators and their connections

Coupled oscillators

- ✎ practical industrial applications models
- ✎ biological activities

Decomposite a complex nonlinear dynamics into unit oscillators and their connections

$$\Downarrow$$

a reduced dynamical system with symmetry

- ✎ synchronization
- ✎ global/local bifurcations

Previous studies

Resistively coupled BVP oscillators

- ✎ Symmetrical connected oscillators have much variety of synchronization modes of limit cycles

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- but **do not have chaos** within reasonable parameter range.

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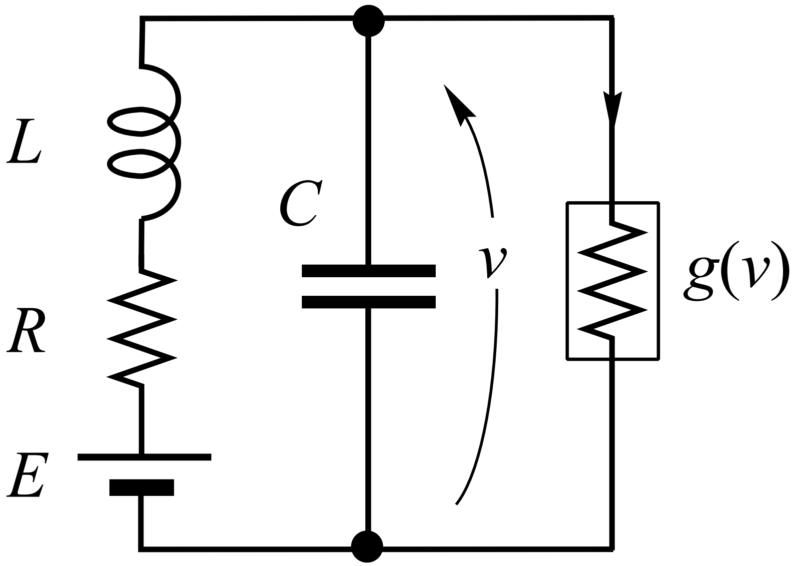
Previous studies

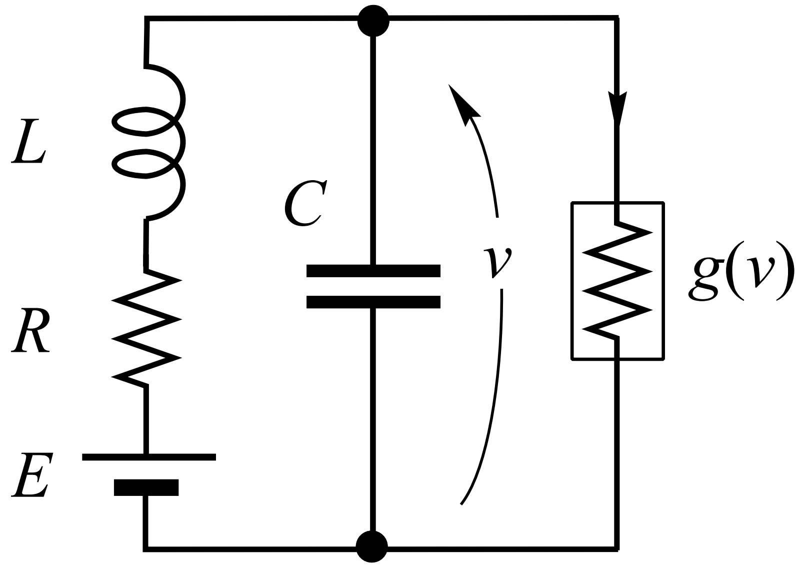
Resistively coupled BVP oscillators

- Symmetrical connected oscillators have much variety of synchronization modes of limit cycles
- but **do not have chaos** within reasonable parameter range.
- since symmetrical properties rather build "mild" dynamics.
- asymmetrical coupling induces **Chaos !**

Single BVP oscillator



Single BVP oscillator

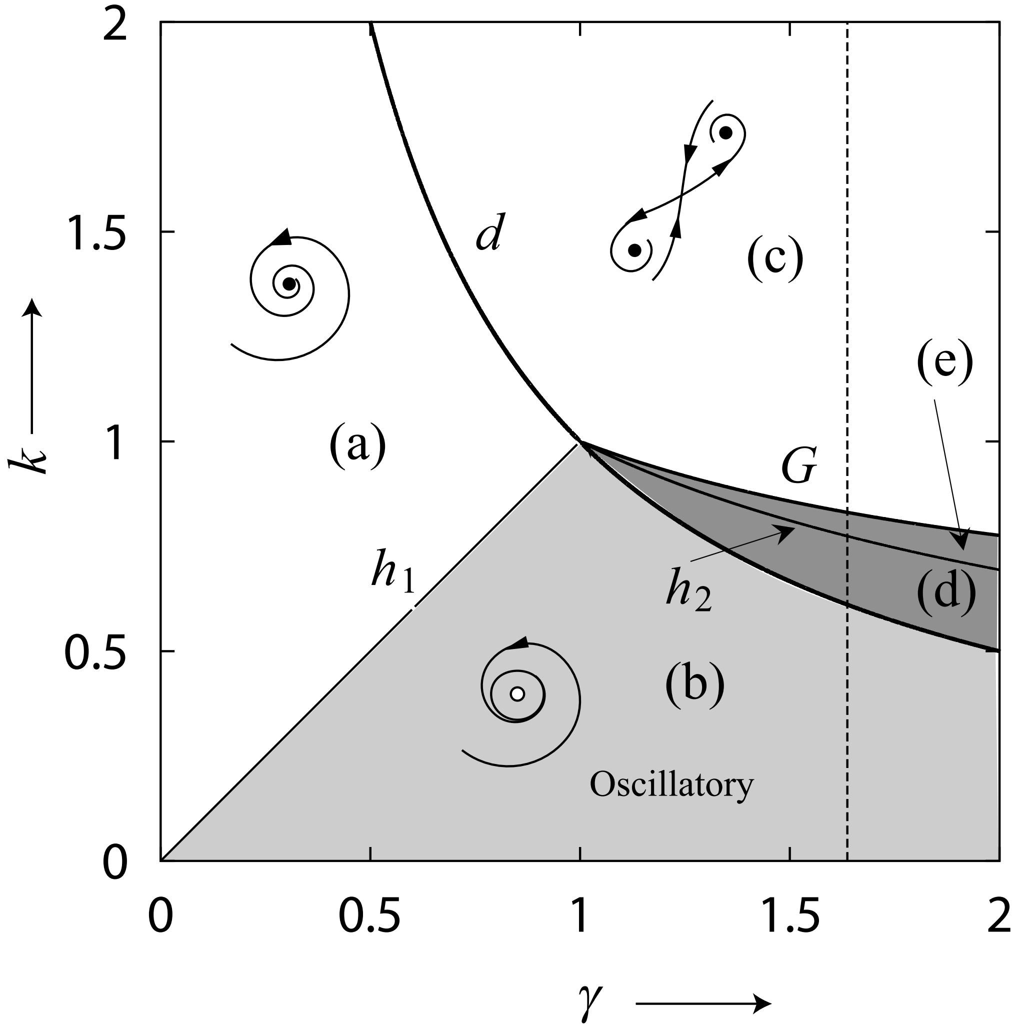


Circuit equation:

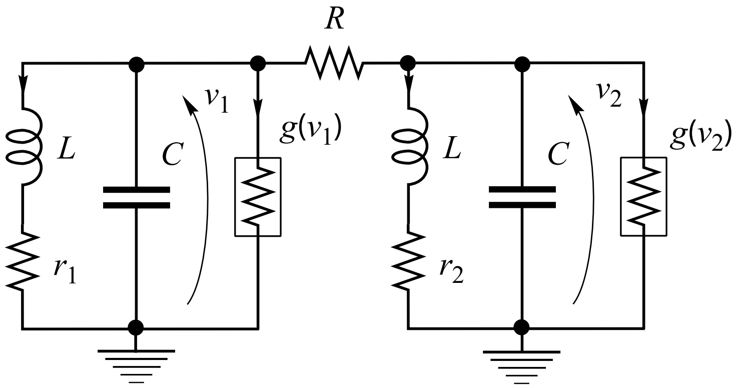
$$C\frac{dv}{dt} = -i - g(v)$$

$$L\frac{di}{dt} = v - ri + E$$

Bifucations in single BVP



Resistively coupled BVP oscillators



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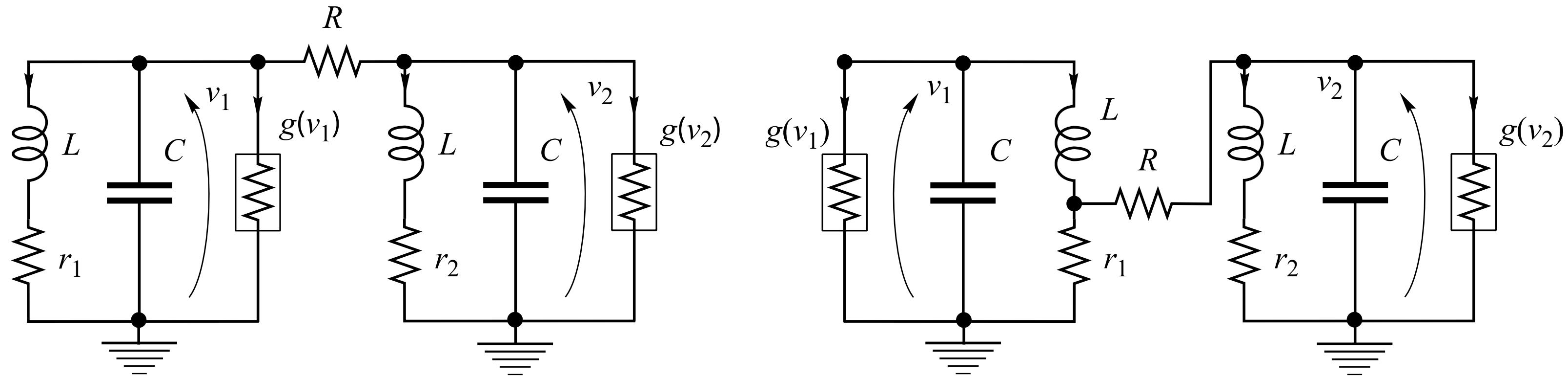
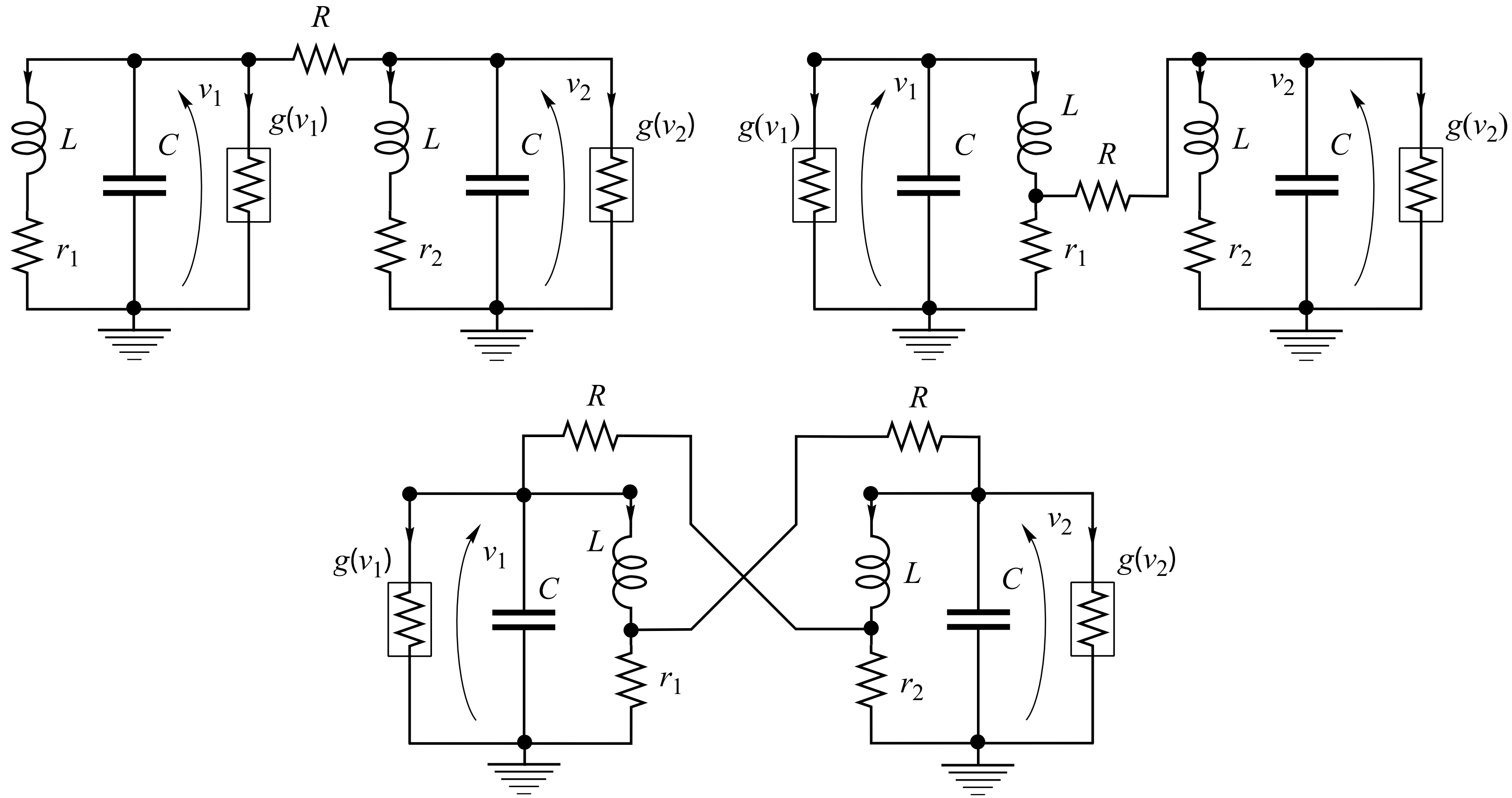
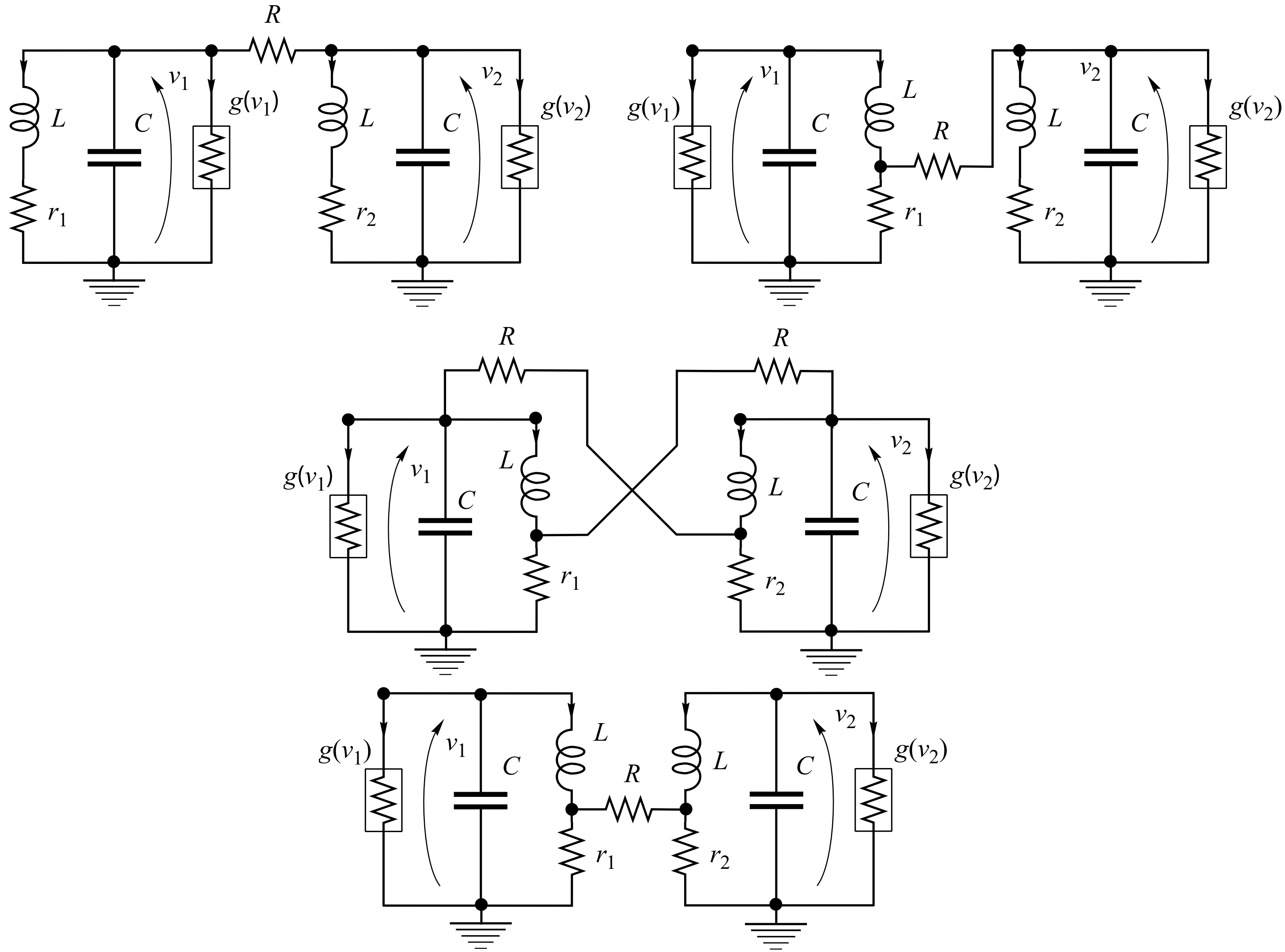


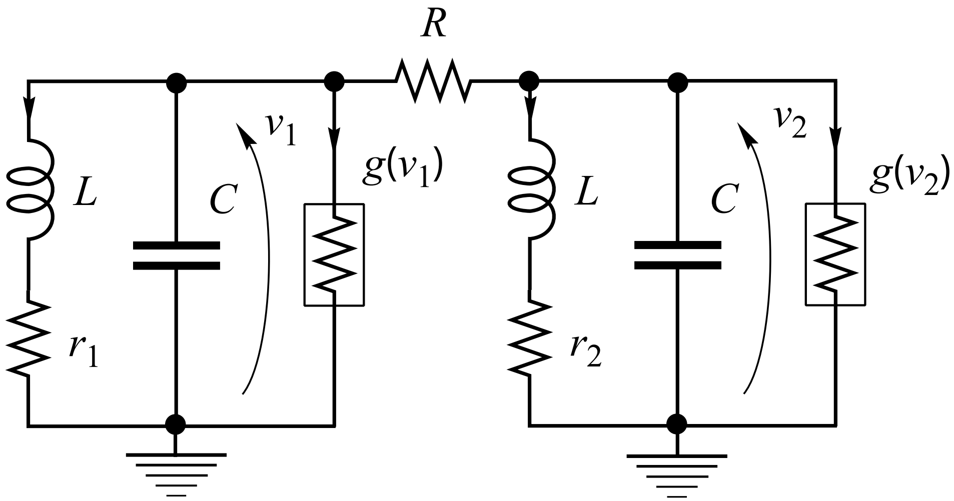
Resistively coupled BVP oscillators

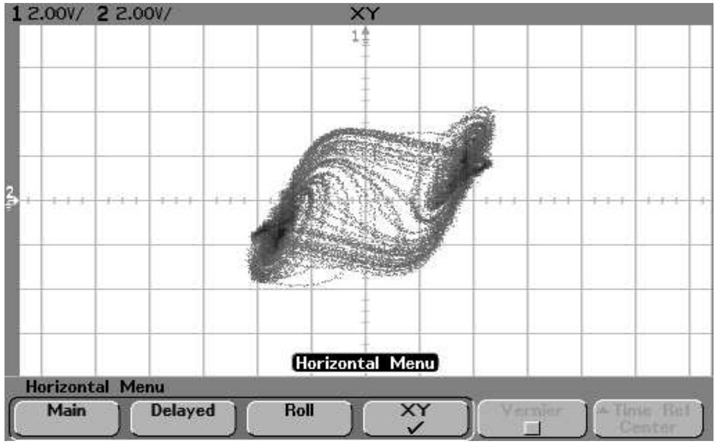


Resistively coupled BVP oscillators



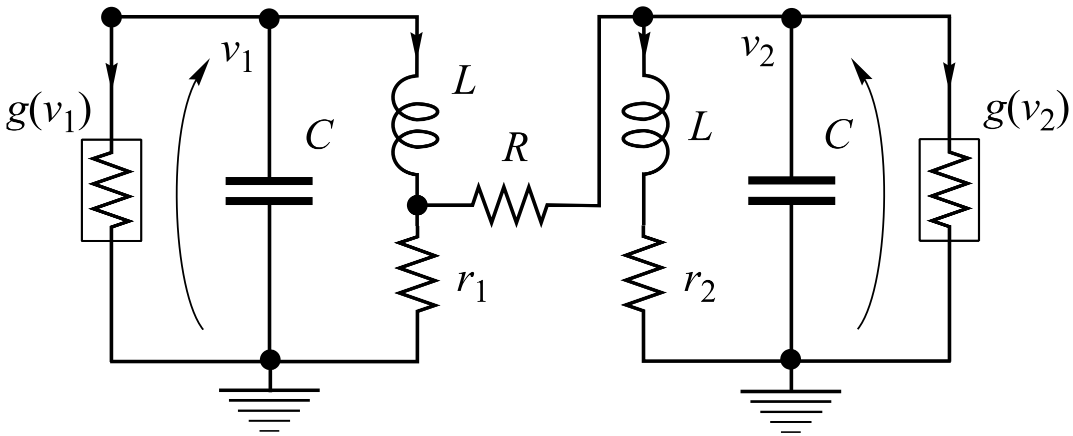
v-v coupled BVP oscillators

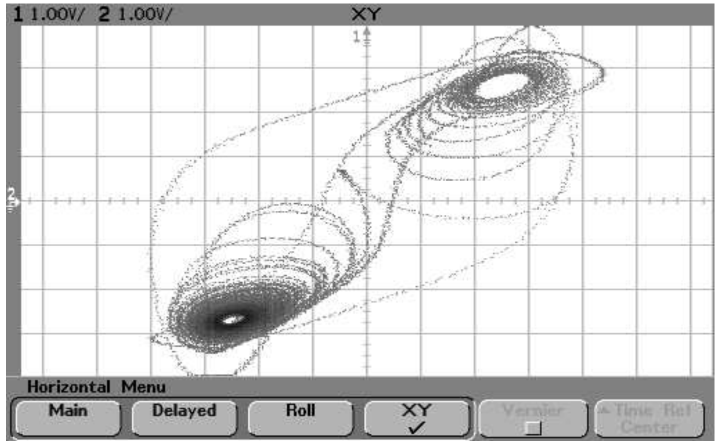




✎ T. Ueta, *et al.*, Strange attractor in resistively coupled BVP oscillators, In Proc. 2001 Int. Conf. on Progress in Nonlinear Science, Russia, July 2001.

v-i coupled BVP oscillators

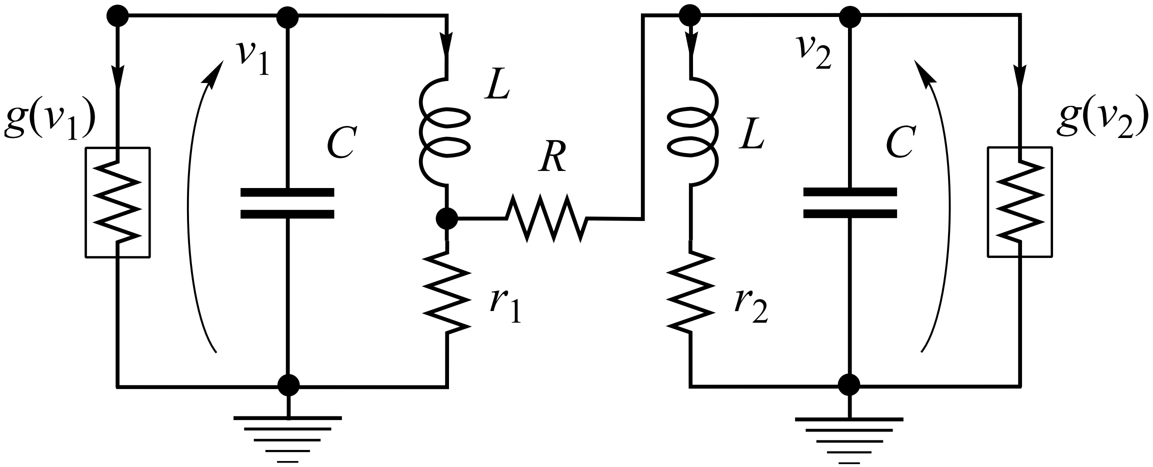




✎ T. Ueta, *et al.*, Bifurcation and Chaos in Asymmetrically Coupled BVP Oscillators, ISCAS 2002, Scottsdale, Arizona, Int. J. Bifurcation and Chaos (to appear)

In this talk. . .

Reducing the v-i coupled circuit



In this talk. . .

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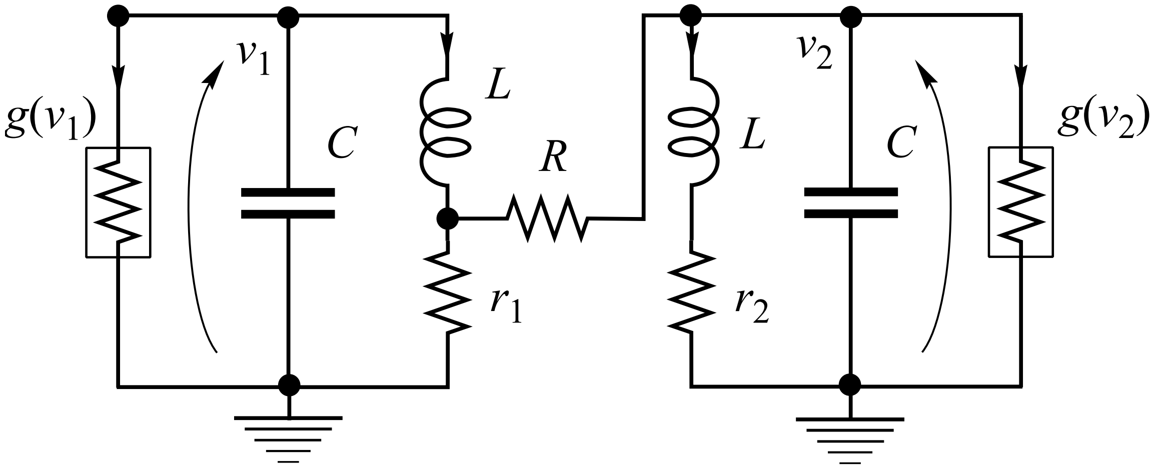
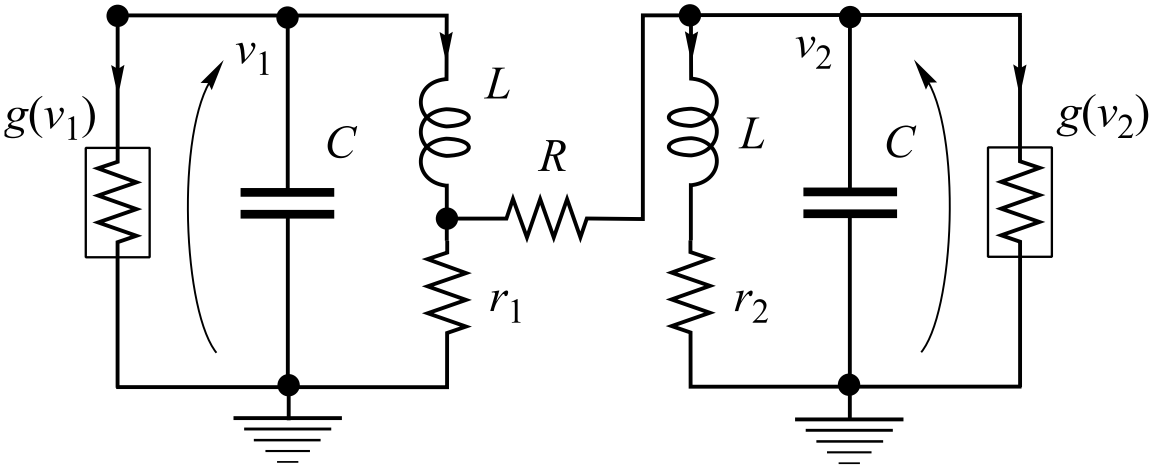


removing $g(v_2)$, letting $r_2 = \infty$, $R = 0$,

In this talk. . .

Reducing the v-i coupled circuit



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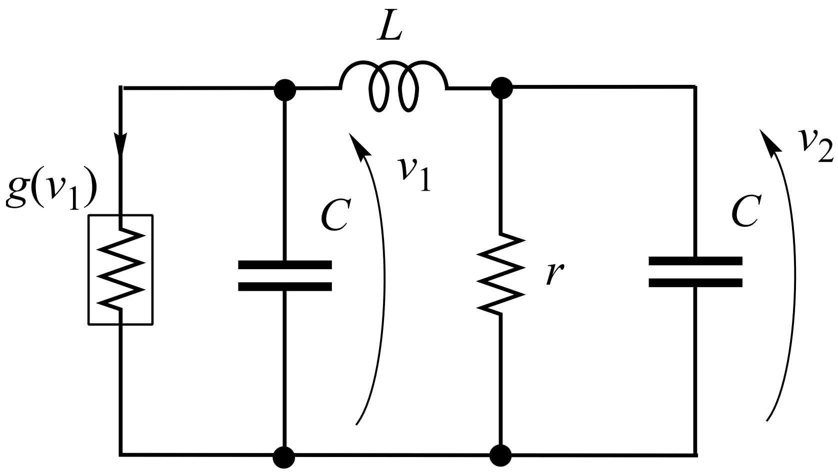
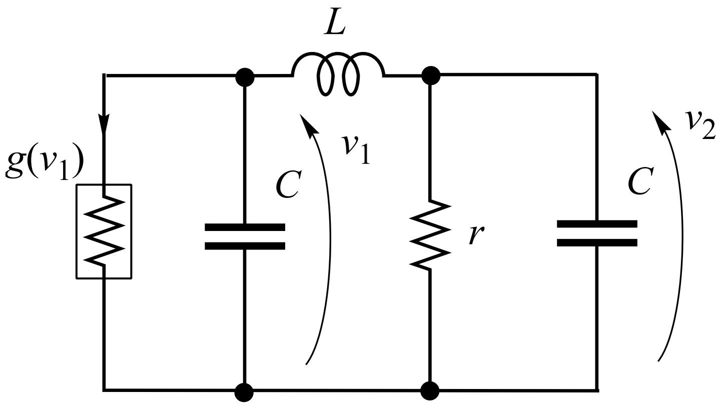
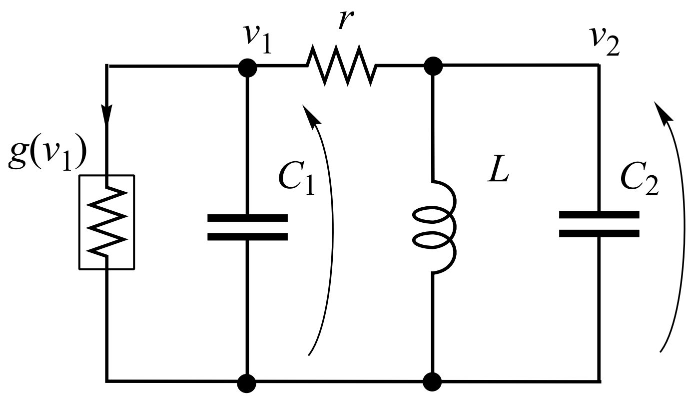


Extended BVP oscillator



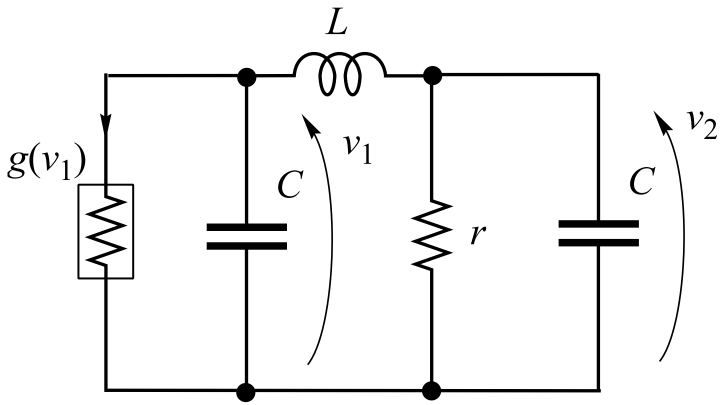
vs



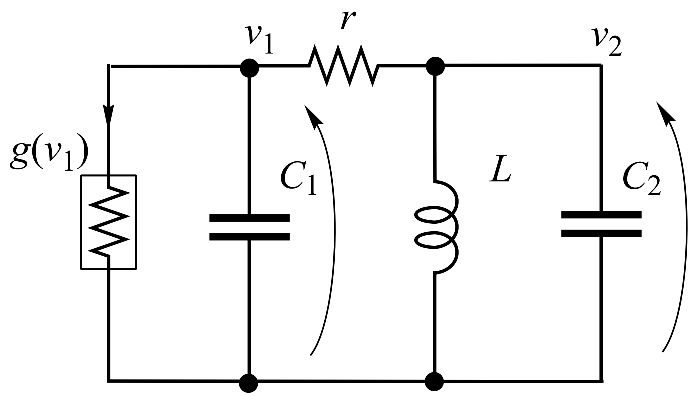
Extended BVP Chua

This circuit have already shown as one of all combinations of L, C, r, $g(v)$. [Chua *et al*, 1992]

Extended BVP oscillator



vs



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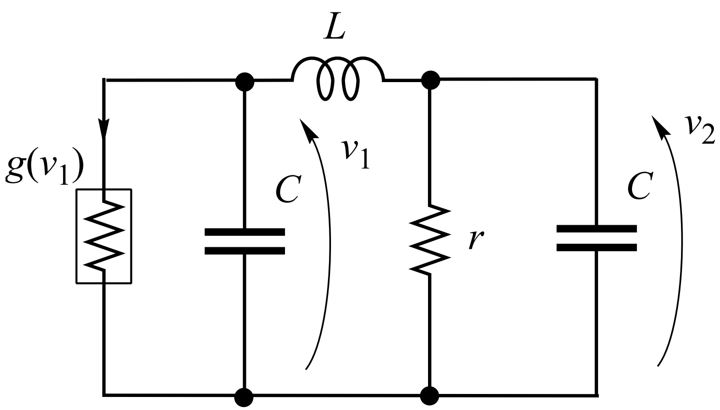
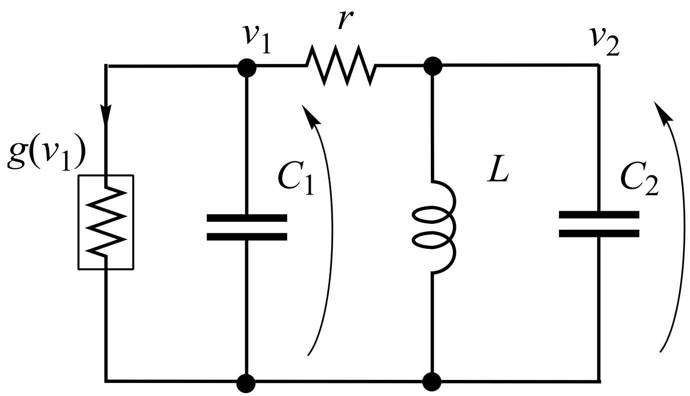
☞But not mentioned these dynamics.

Extended BVP oscillator



vs



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☞But not mentioned these dynamics.

☞Naturally induced from coupled system!

Circuit equation

$$
\begin{cases}
C\dfrac{dv_1}{dt} = -i - g(v_1) \\
C\dfrac{dv_2}{dt} = i - \dfrac{v_2}{r} \\
L\dfrac{di}{dt} = v_1 - v_2
\end{cases}
$$

Circuit equation

$$\begin{cases} C\dfrac{dv_1}{dt} = -i - g(v_1) \\ C\dfrac{dv_2}{dt} = i - \dfrac{v_2}{r} \\ L\dfrac{di}{dt} = v_1 - v_2 \end{cases}$$

$$g(v) = -a \tanh bv, \quad \tau = t/\sqrt{LC},$$

$$\gamma = ab\sqrt{\frac{L}{C}}, \quad \delta = \frac{1}{r}\sqrt{\frac{L}{C}}$$

$$x = \frac{v_1}{a}\sqrt{\frac{C}{L}}, \quad y = \frac{v_2}{a}\sqrt{\frac{C}{L}}, \quad z = \frac{i}{a}.$$

Normalized equation

$$\begin{aligned}\dot{x} &= -z + \tanh \gamma x \\ \dot{y} &= z - \delta y \\ \dot{z} &= x - y\end{aligned}$$

Equivalently,

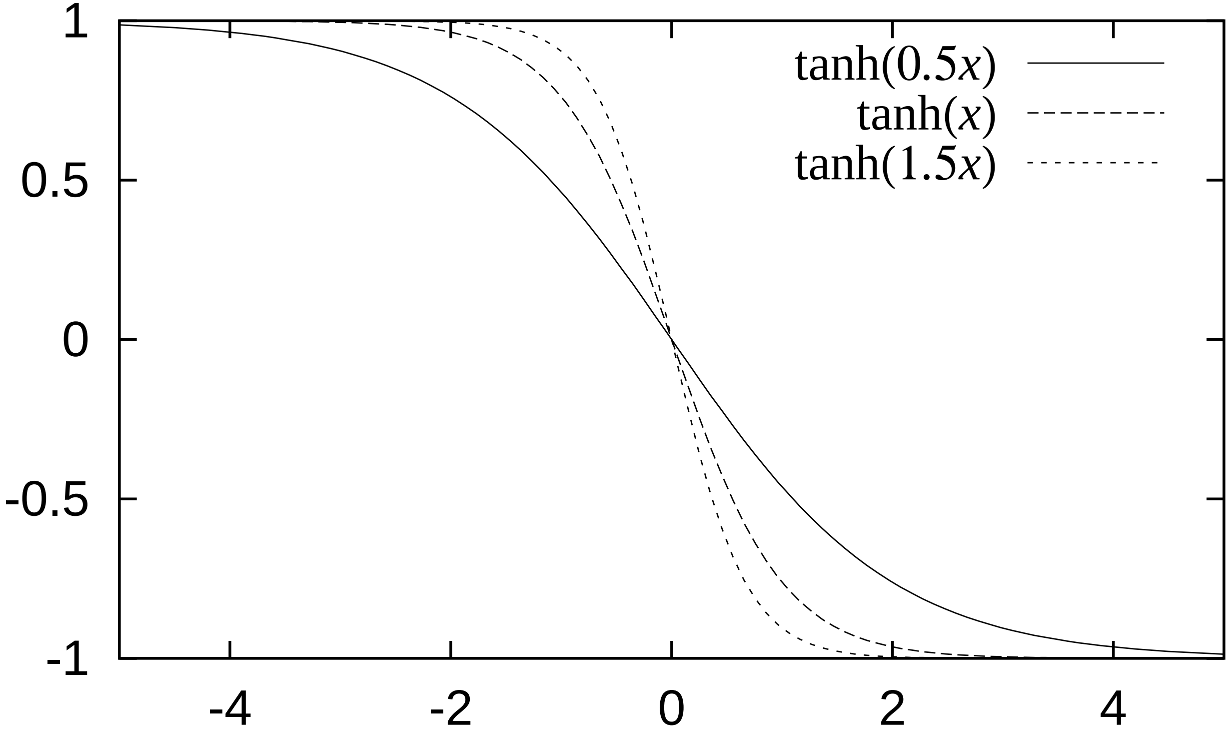
$$\dddot{x} + \alpha(x)\ddot{x} + \beta(x, \dot{x})\dot{x} + \delta x - \tanh \gamma x = 0$$

where,

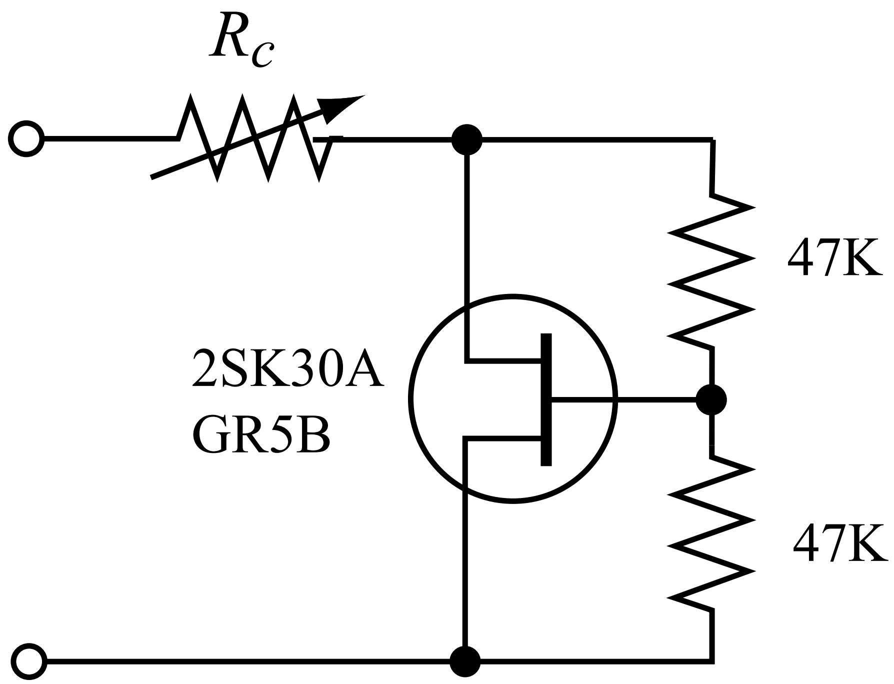
$$\alpha(x) = \delta - \gamma \operatorname{sech}^2 \gamma x$$
$$\beta(x, \dot{x}) = 2 + 2\gamma^2 \operatorname{sech}^2 \gamma x \tanh \gamma x \dot{x} - \delta\gamma \operatorname{sech}^2 \gamma x$$

$$g(v) = -a \tanh bv$$

γ-sensitivity:

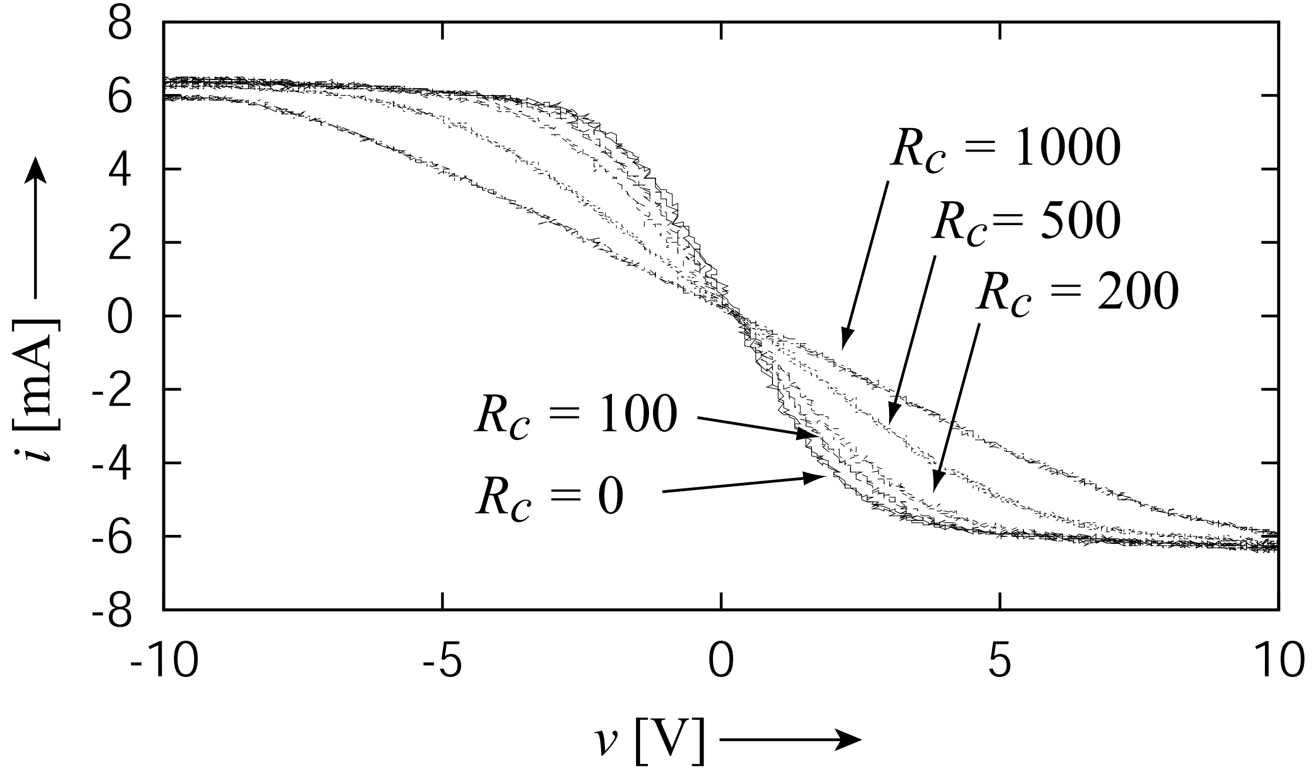


Implementation of $g(v)$



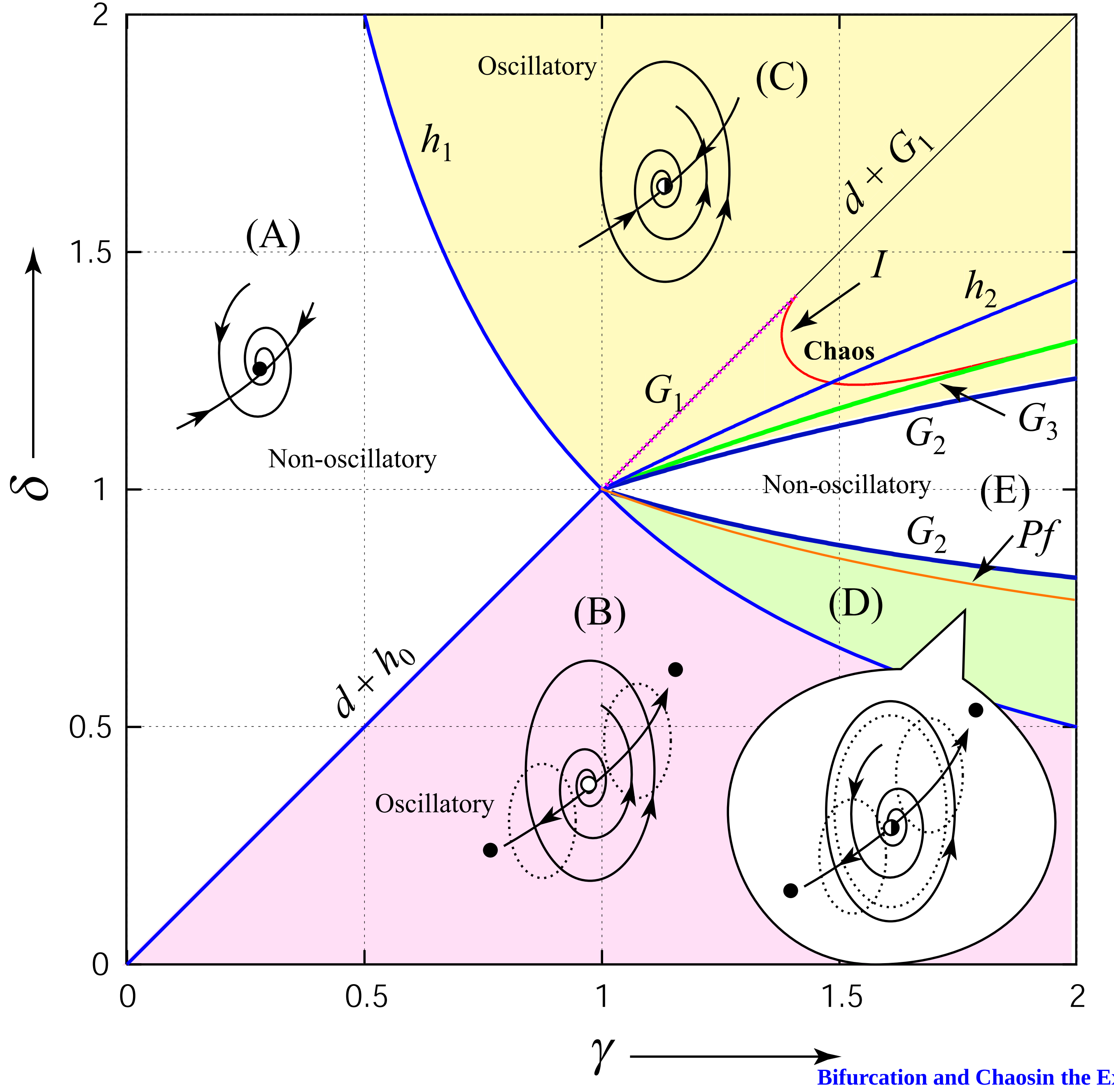
The output of an inverter driving this circuit can realize $-a\tanh bv$.

Experimental measurement

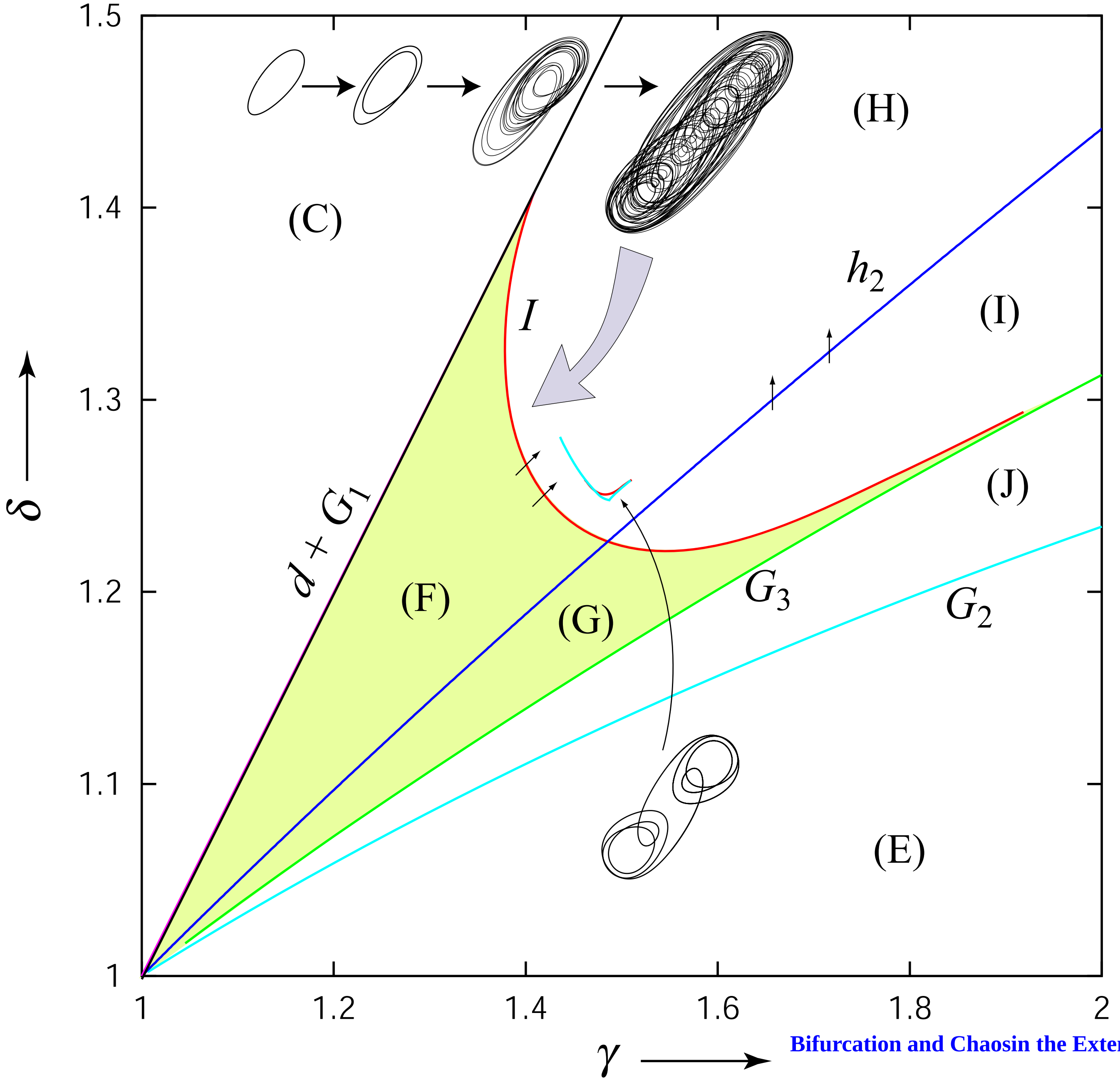


γ can be controlled by R_c.

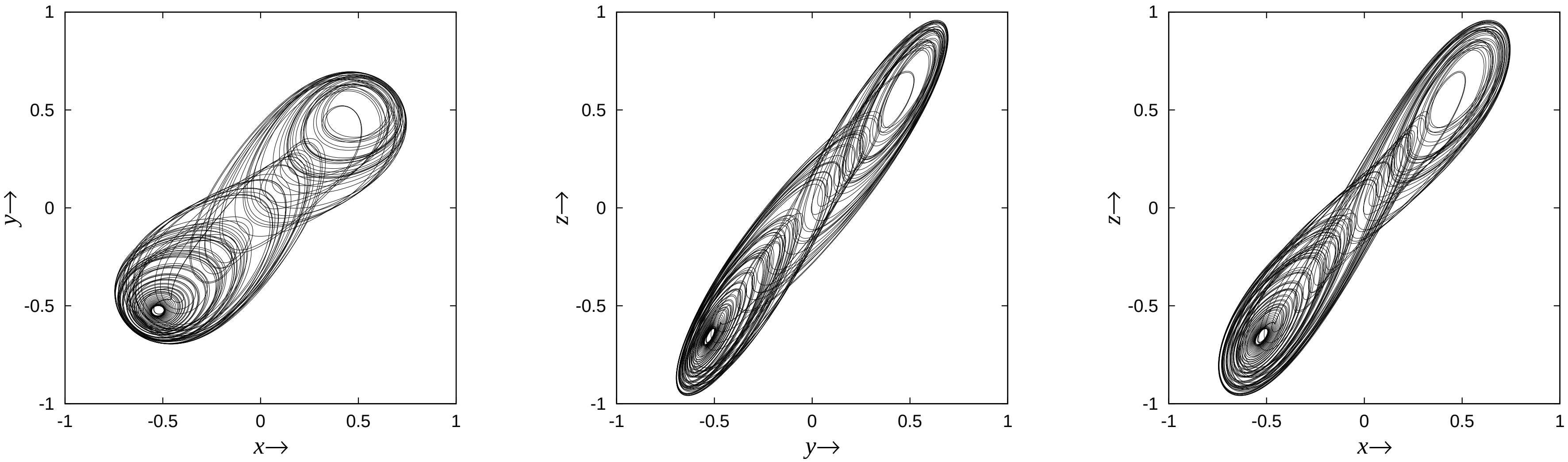
Bifurcation diagram



Bifurcation diagram (magnified)

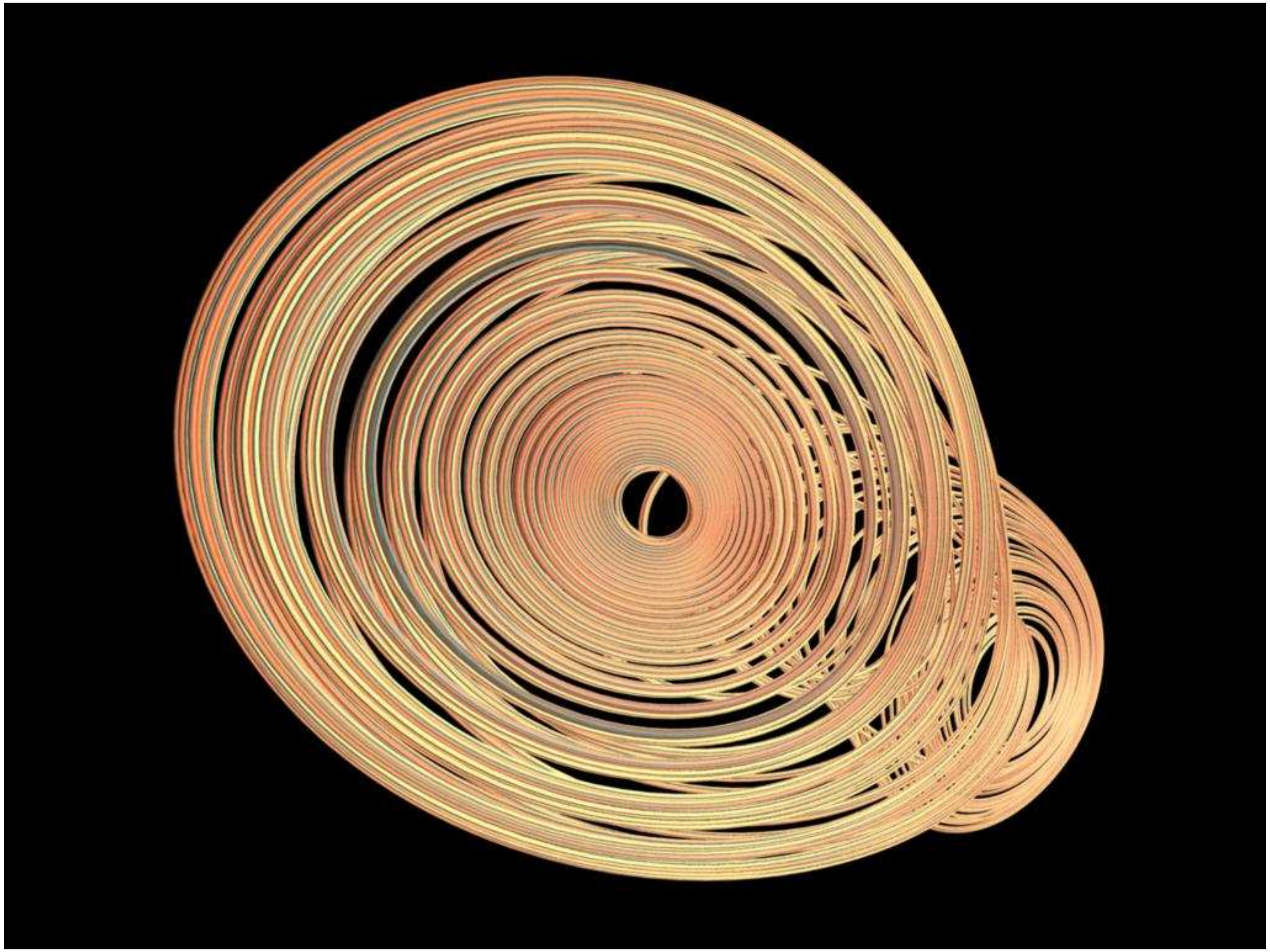


Double scroll

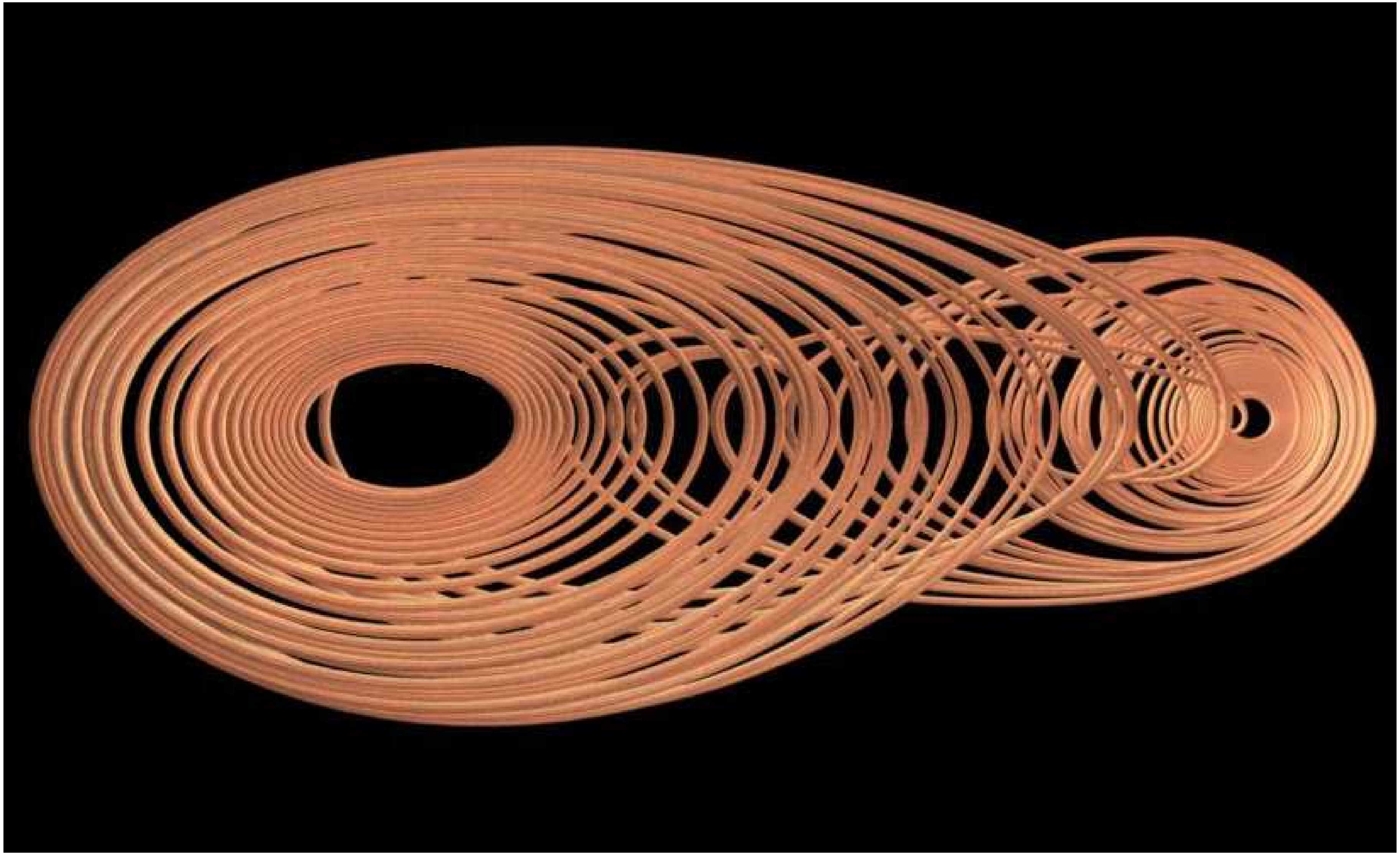


x-y y-z x-z

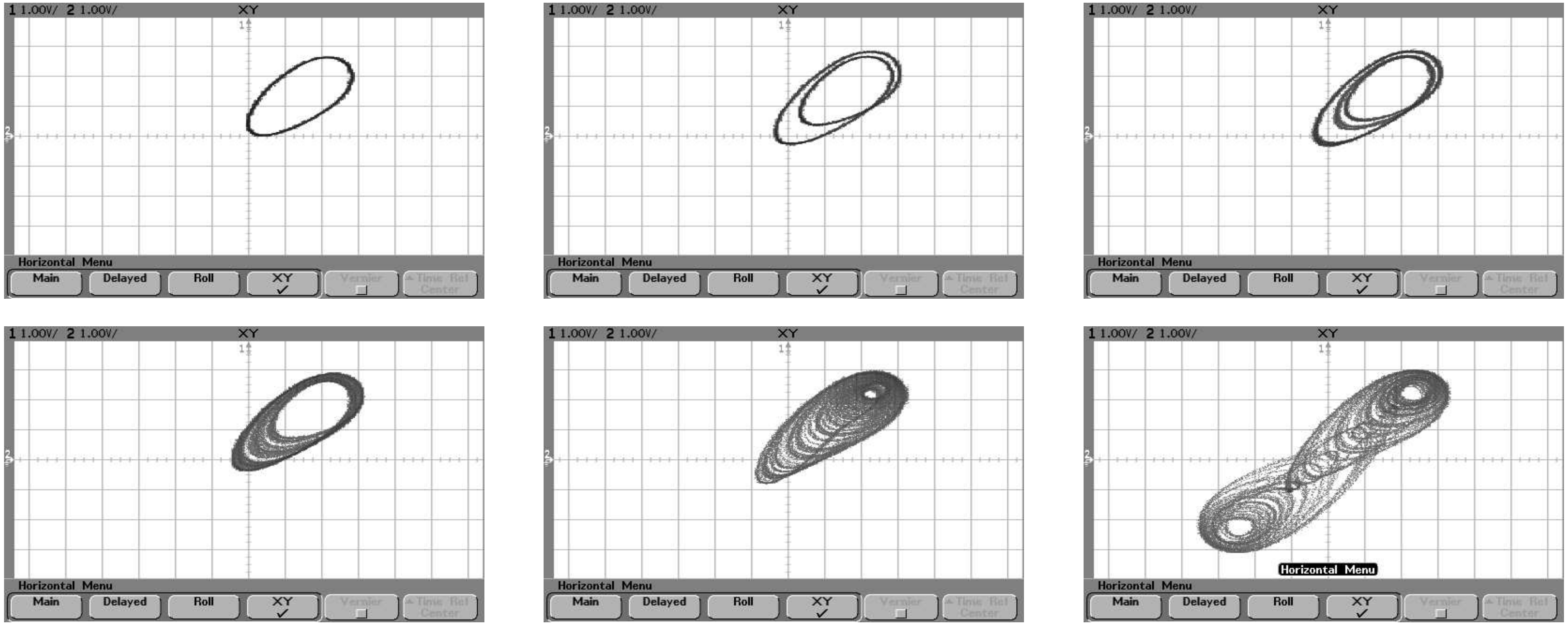
Perspectives



Perspectives

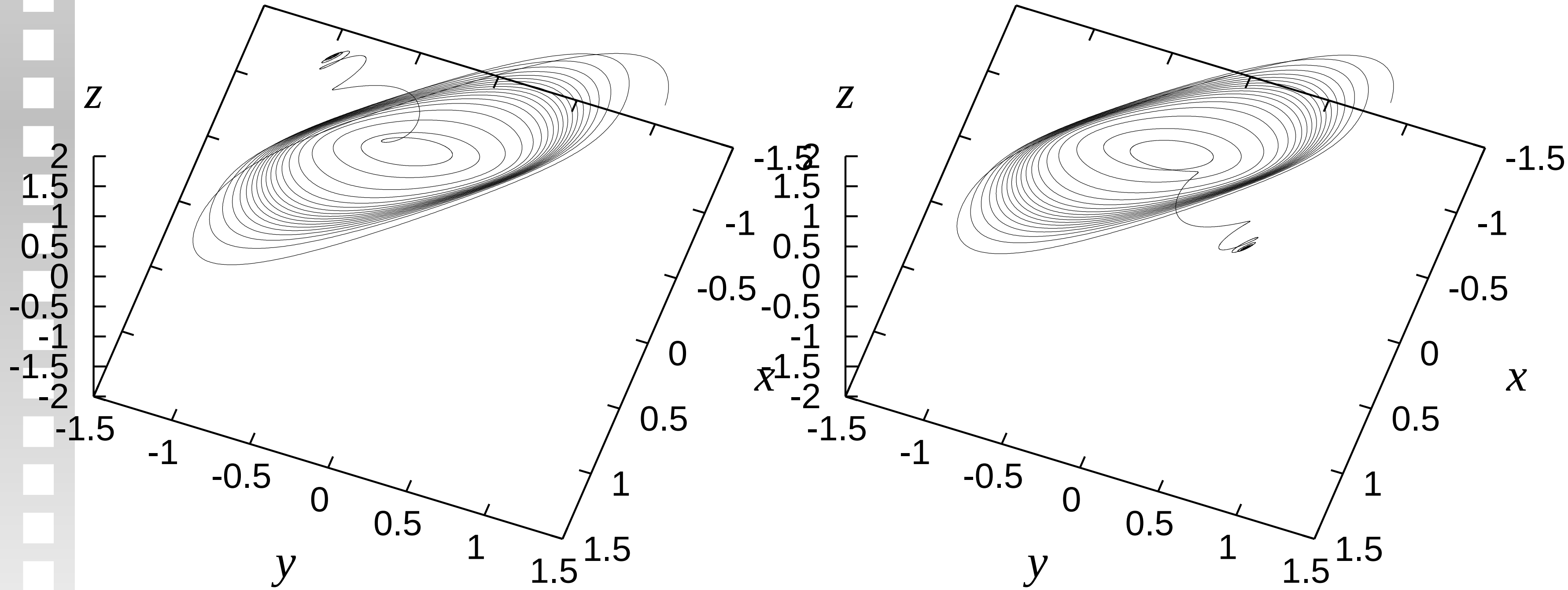


Laboratory experiments

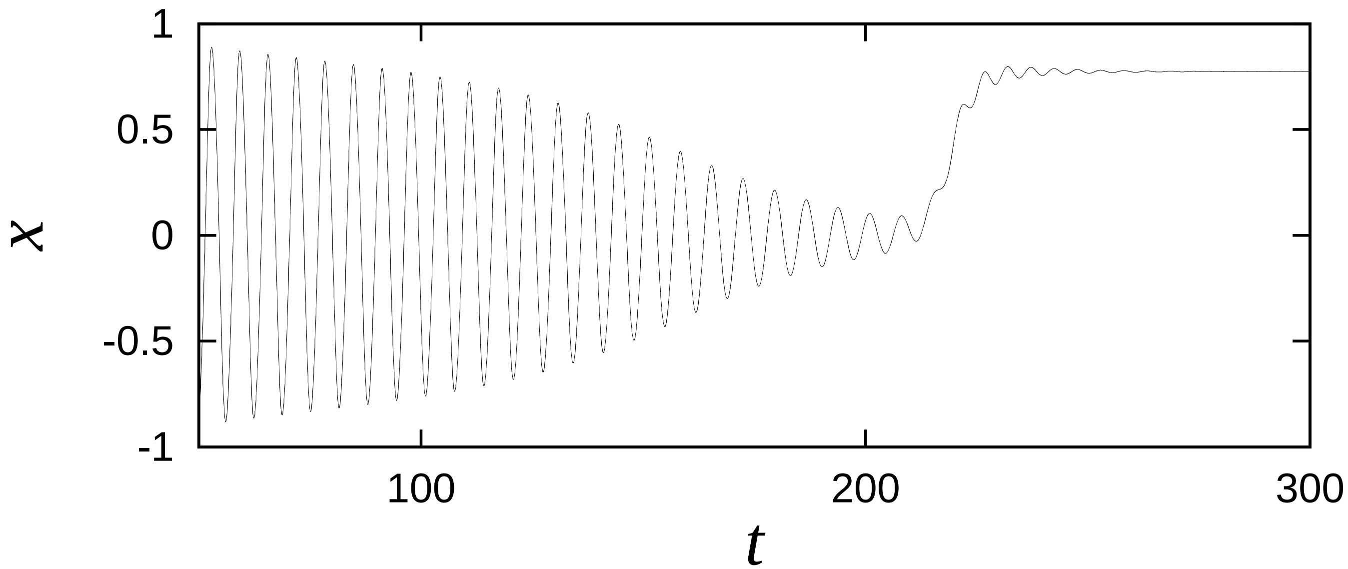


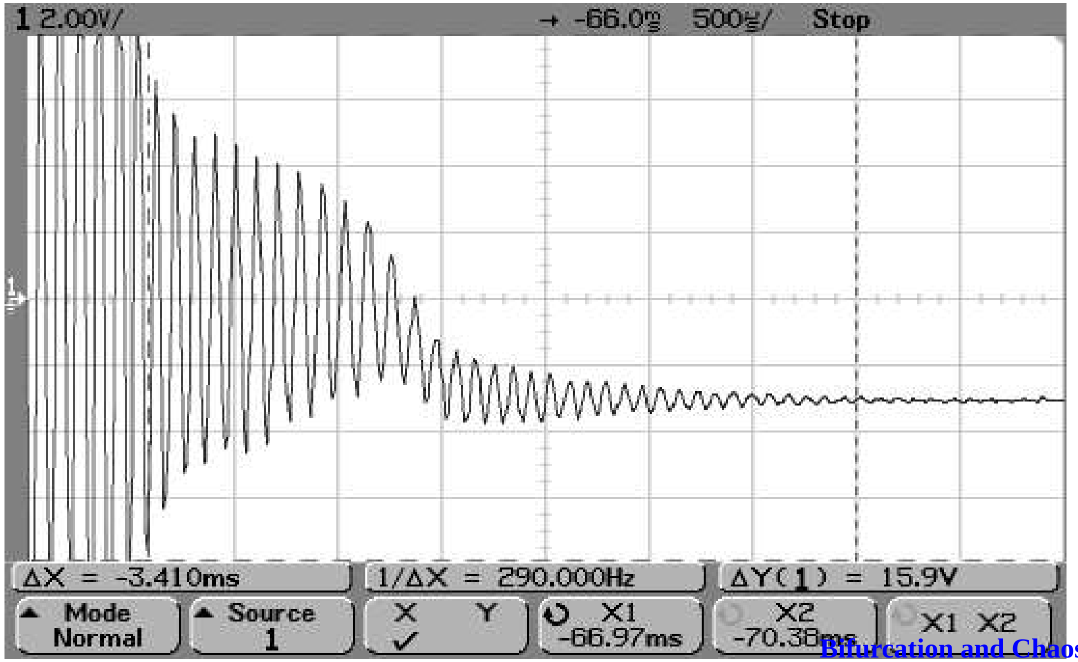
By changing an serial resistance in $g(v)$, $r = 467[\Omega]$.

Jack-in-the-box phenomenon

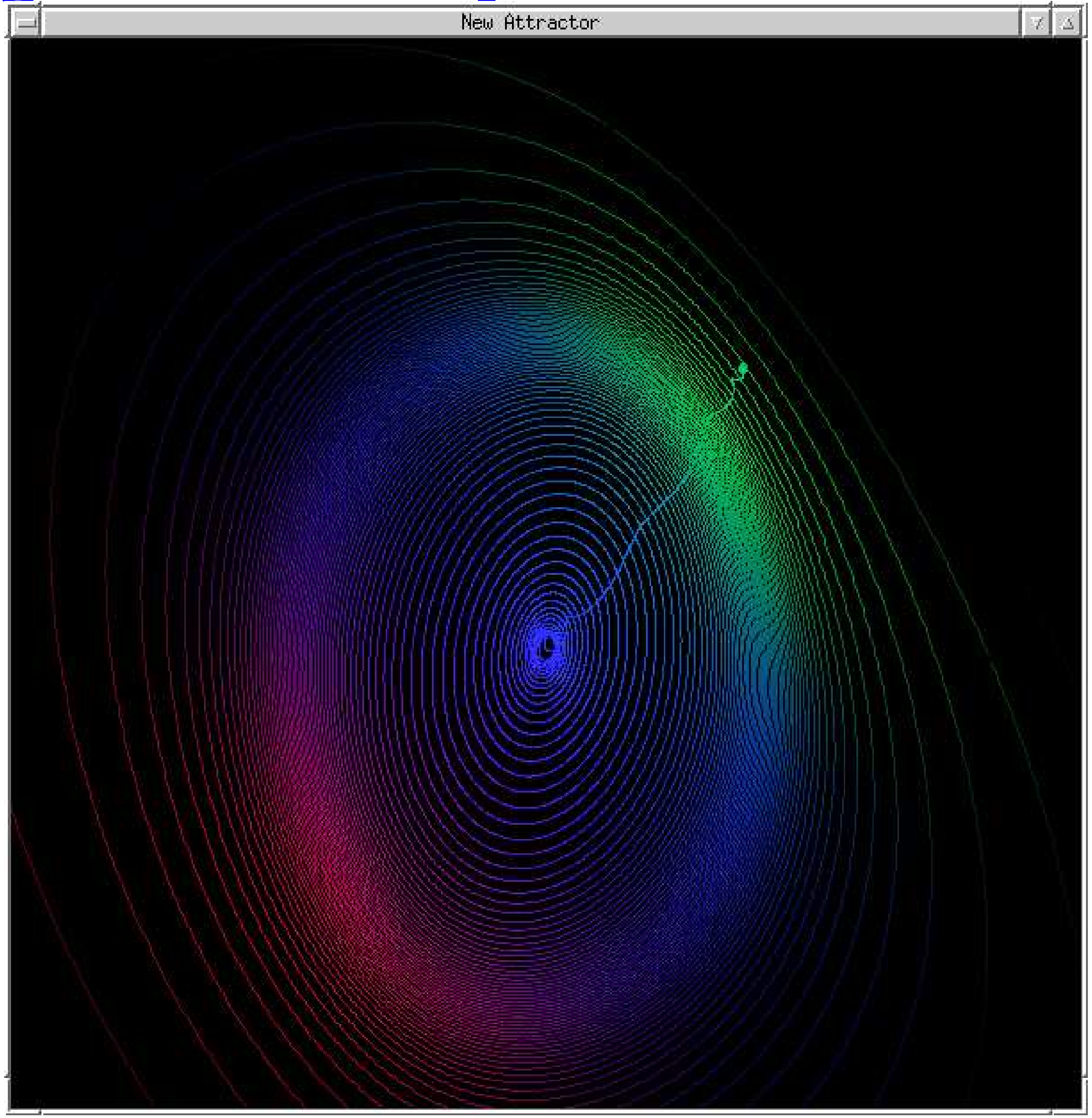


Jack-in-the-box phenomenon(2)



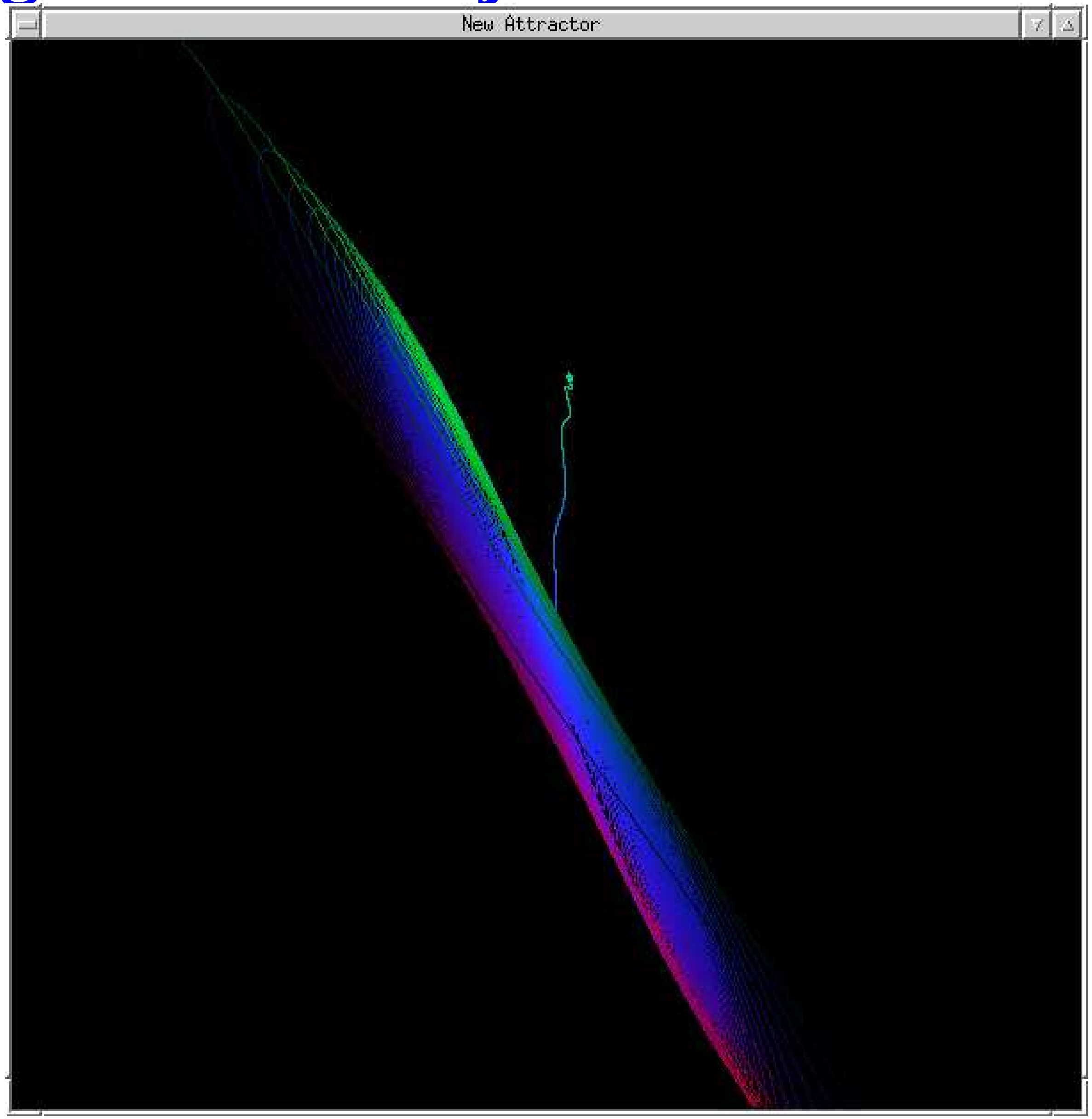


Long transient of jack-in-the-box



$$(x_0, y_0, z_0) = (3, 5, 8)$$

Long transient of jack-in-the-box



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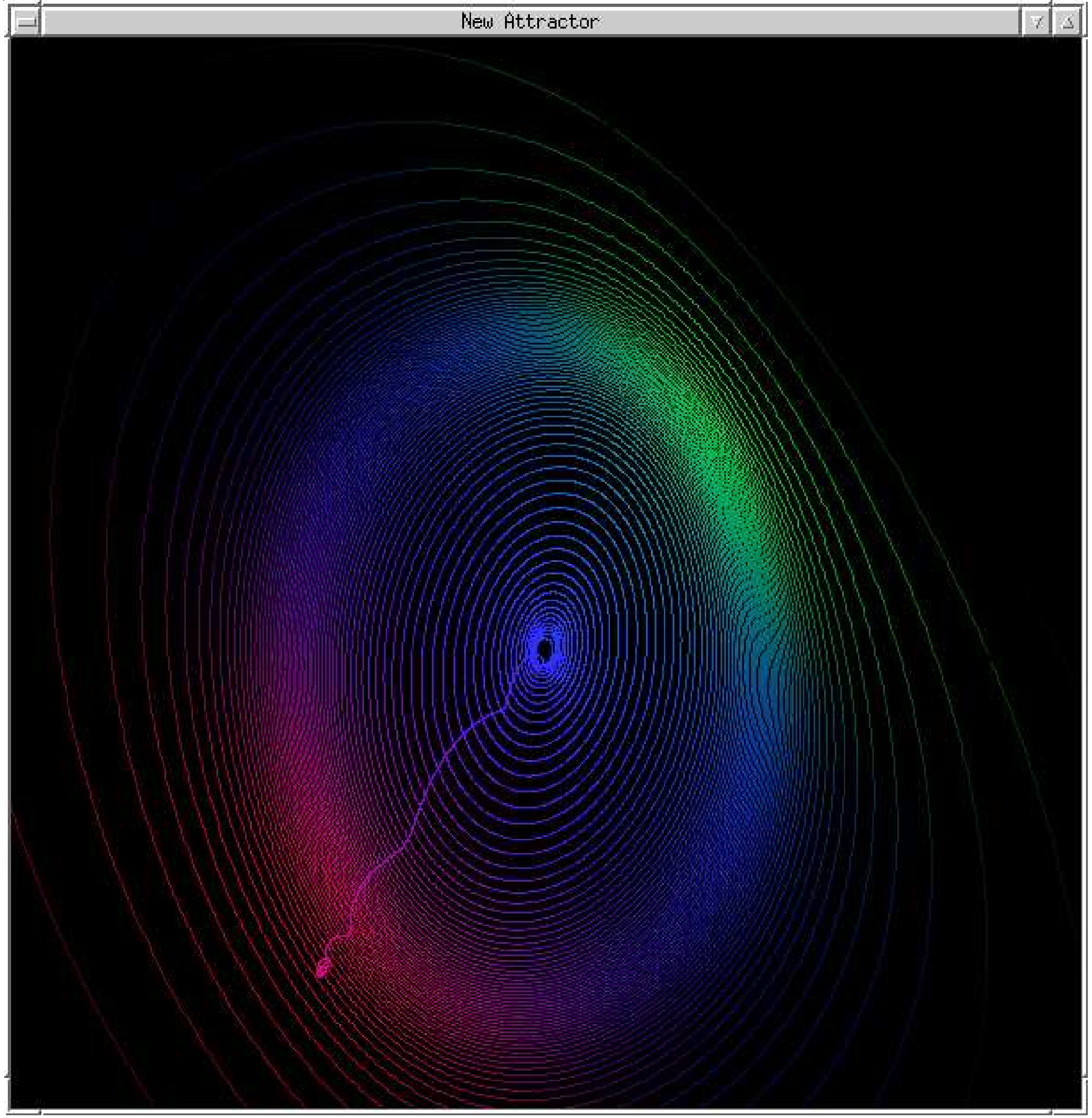
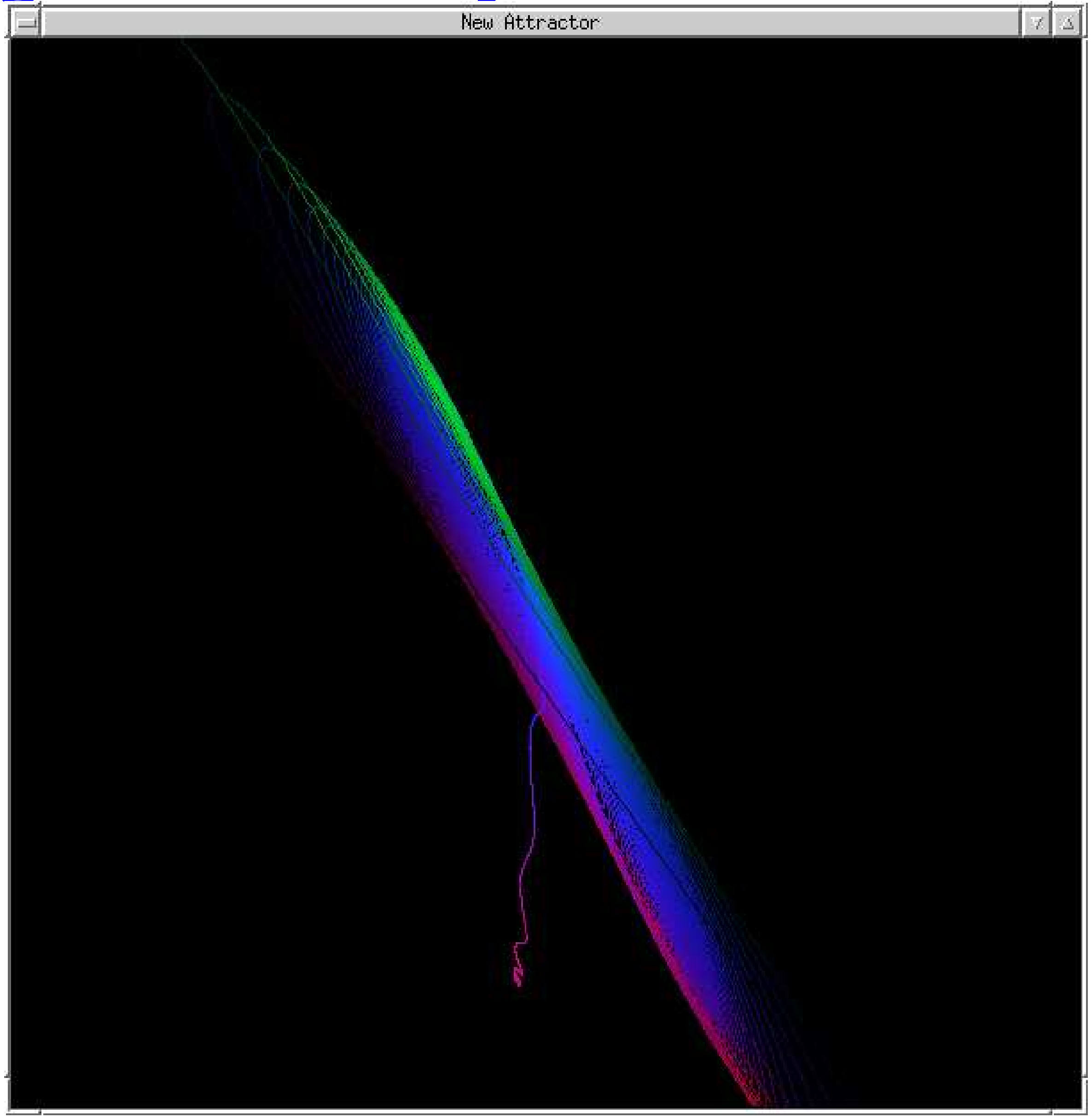


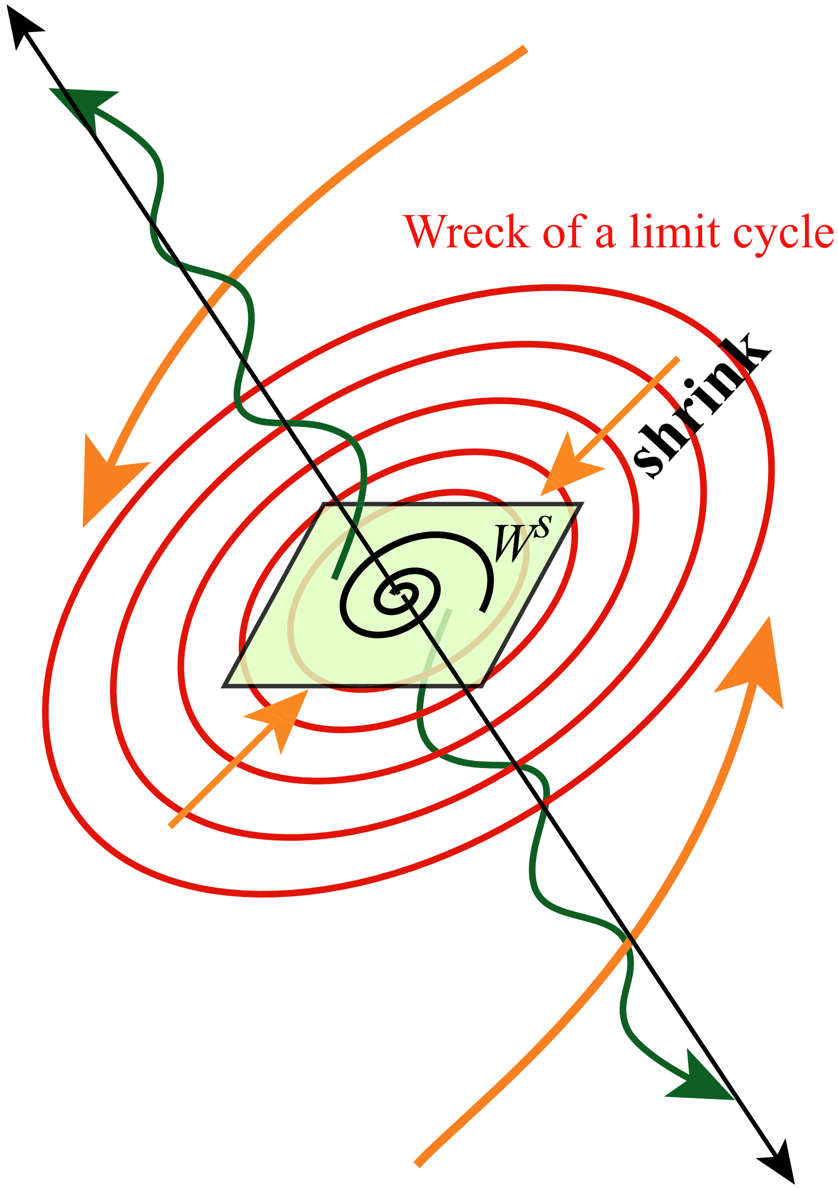
$$(x_0, y_0, z_0) = (3.00001, 5, 8)$$

Long transient of jack-in-the-box

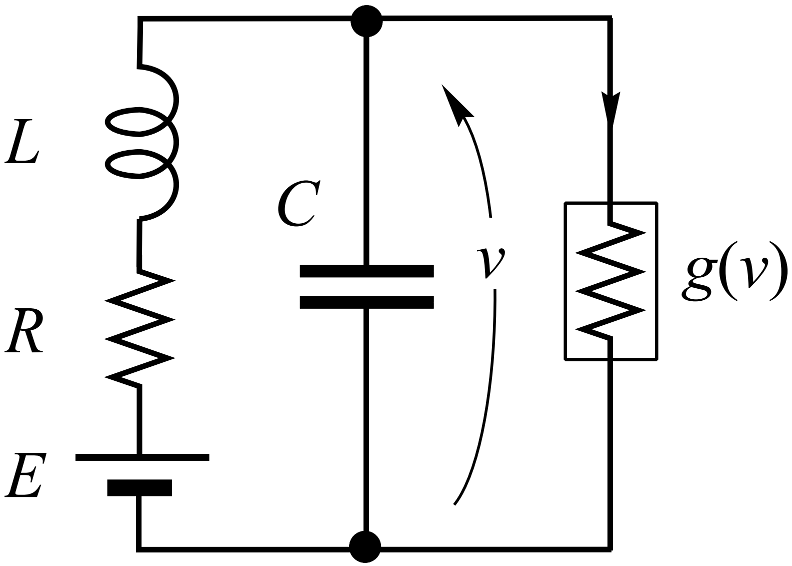


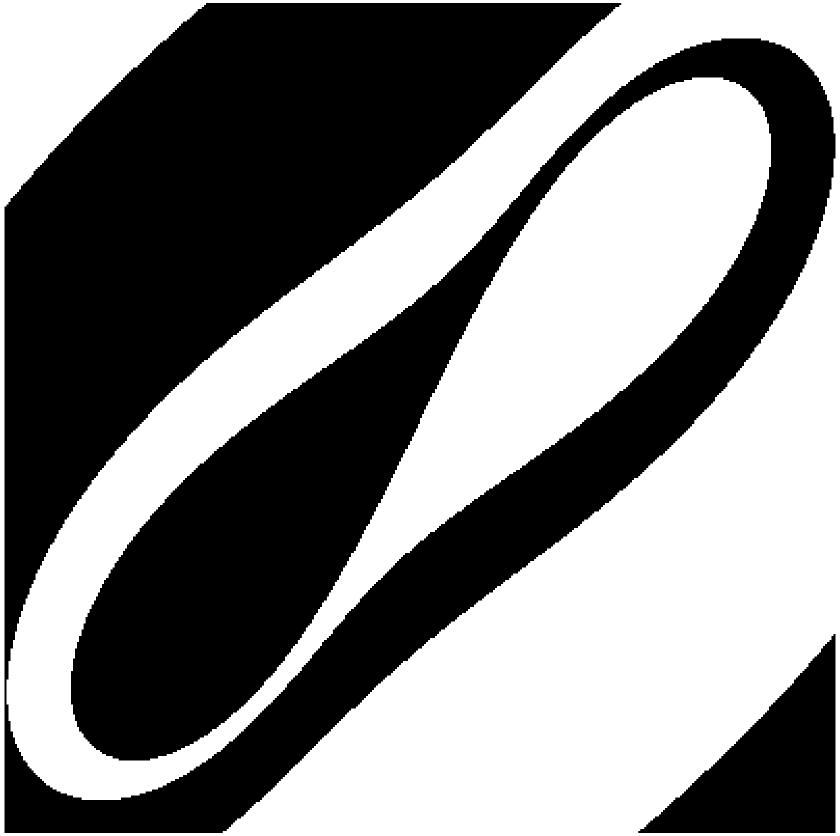
$$(x_0, y_0, z_0) = (3.00001, 5, 8)$$

Generation of jack-in-the-box

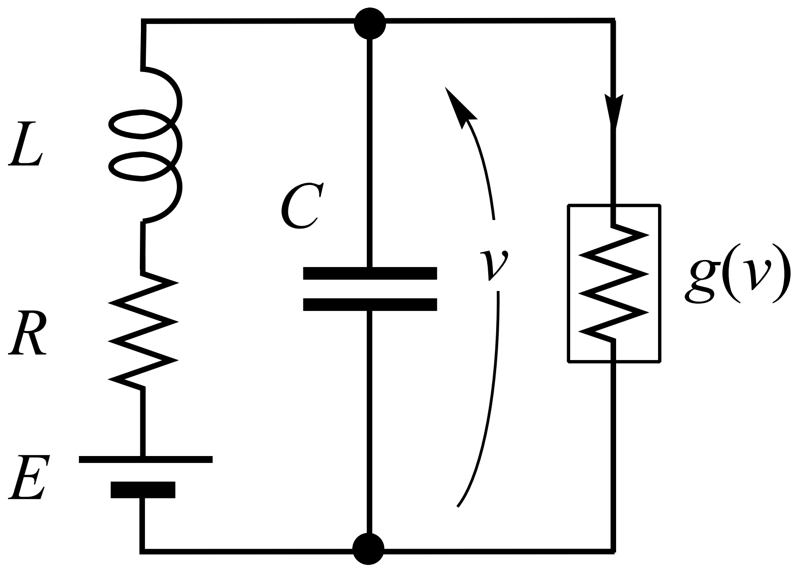


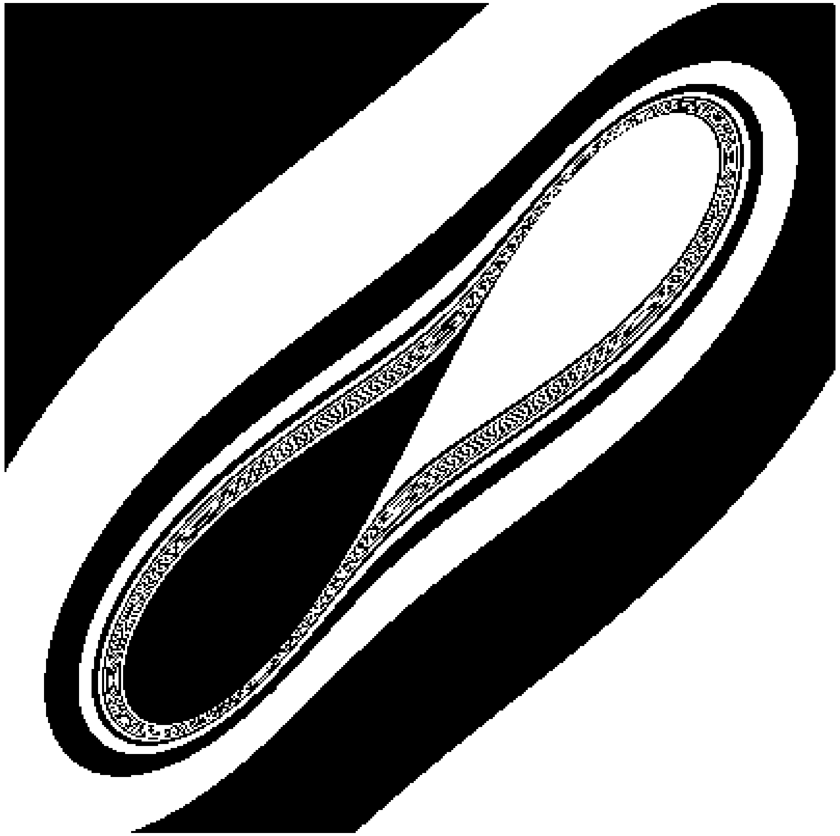
Basin boundary in single BVP



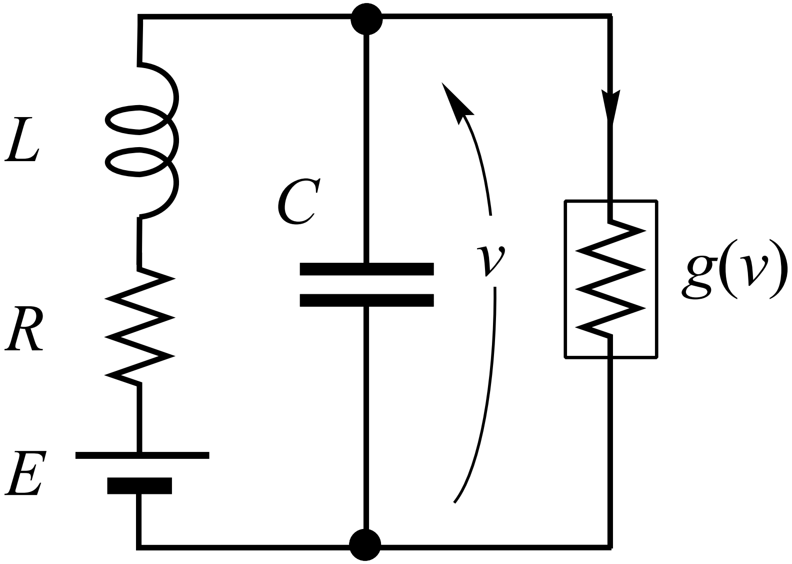


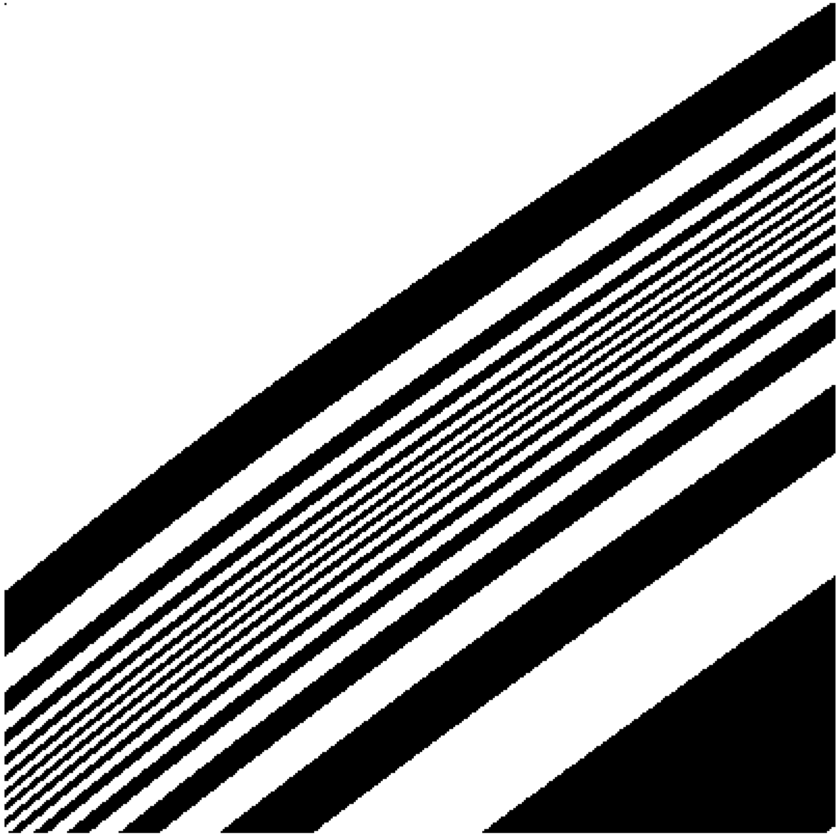
Basin boundary in single BVP





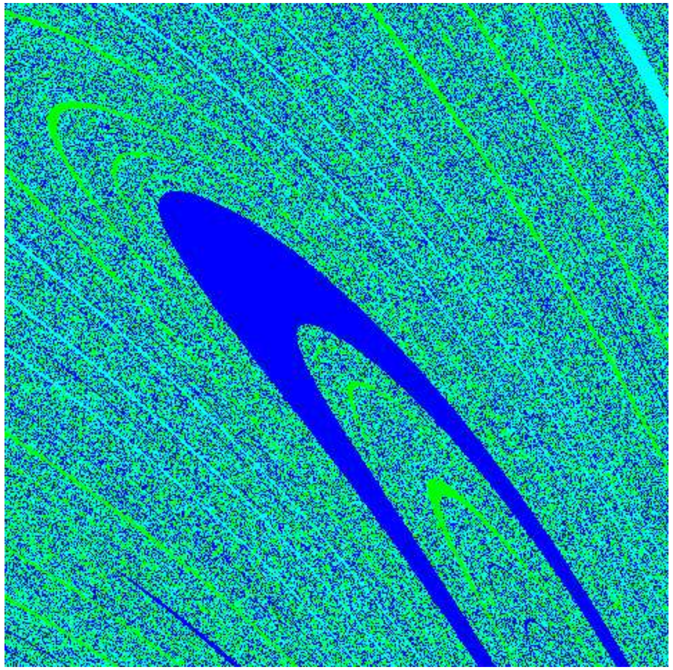
Basin boundary in single BVP





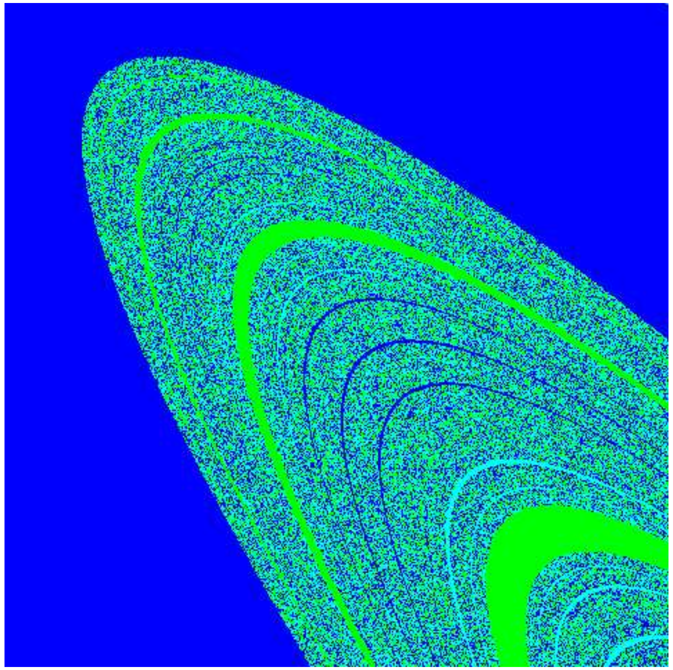
Each basin is continuously separable.

Basin boundary of a 2D map



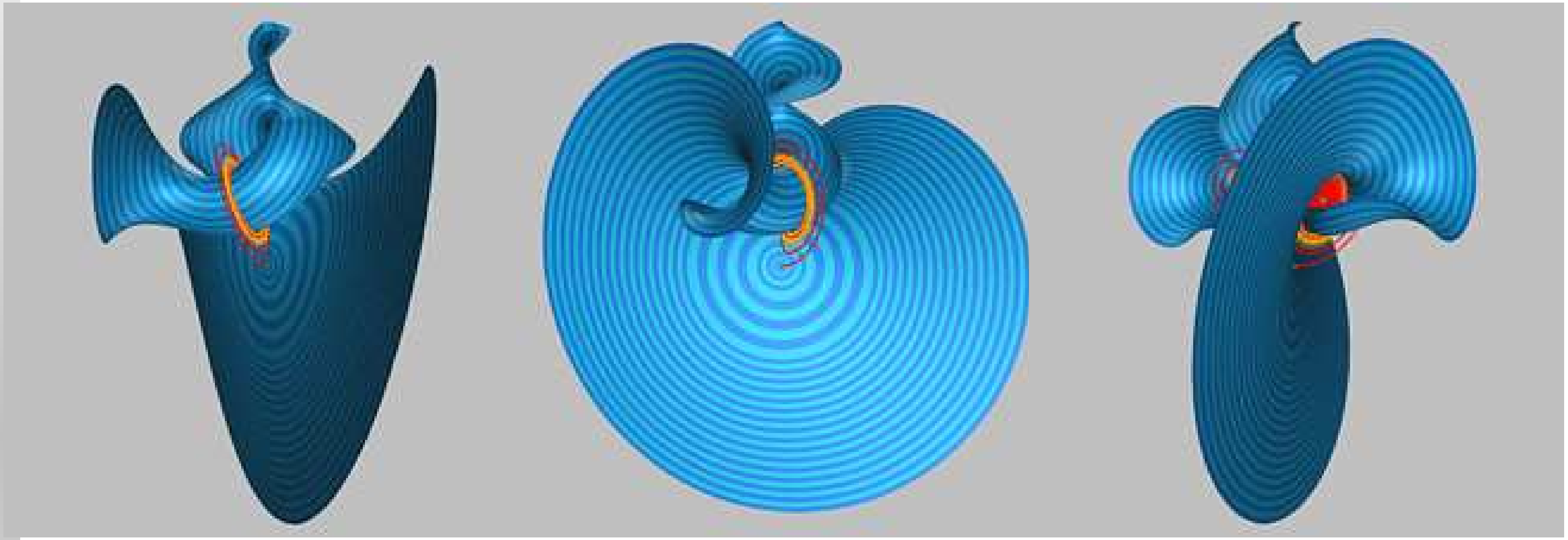
$\Rightarrow$

magnify



Fractal boundary (Mille-feuille structure) is also separable.

Lorenz system



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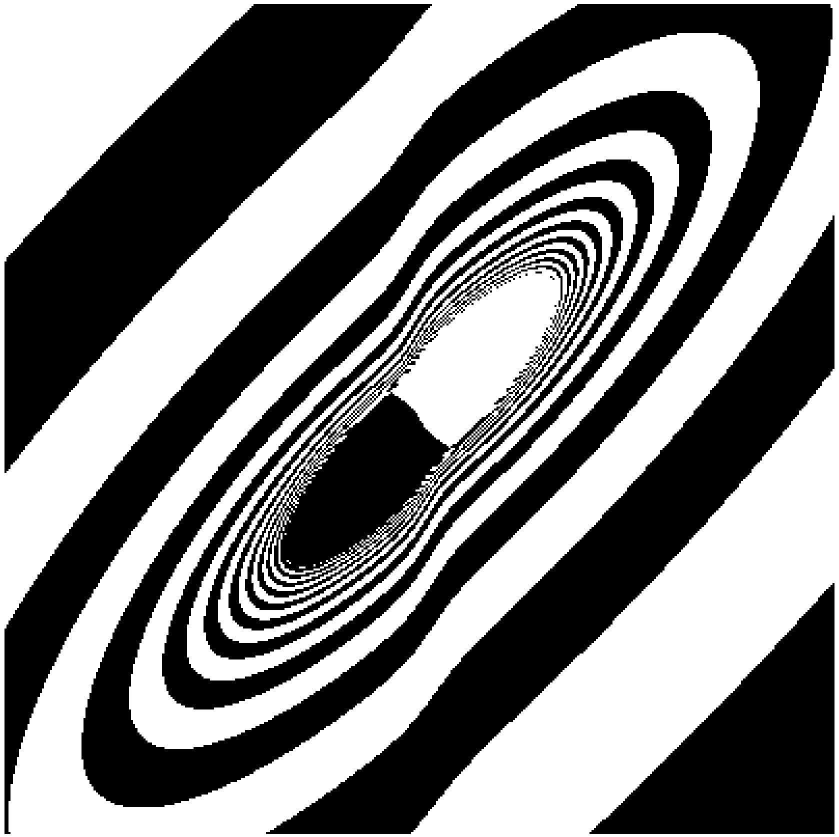
Stable manifold of the origin forms a "surface."

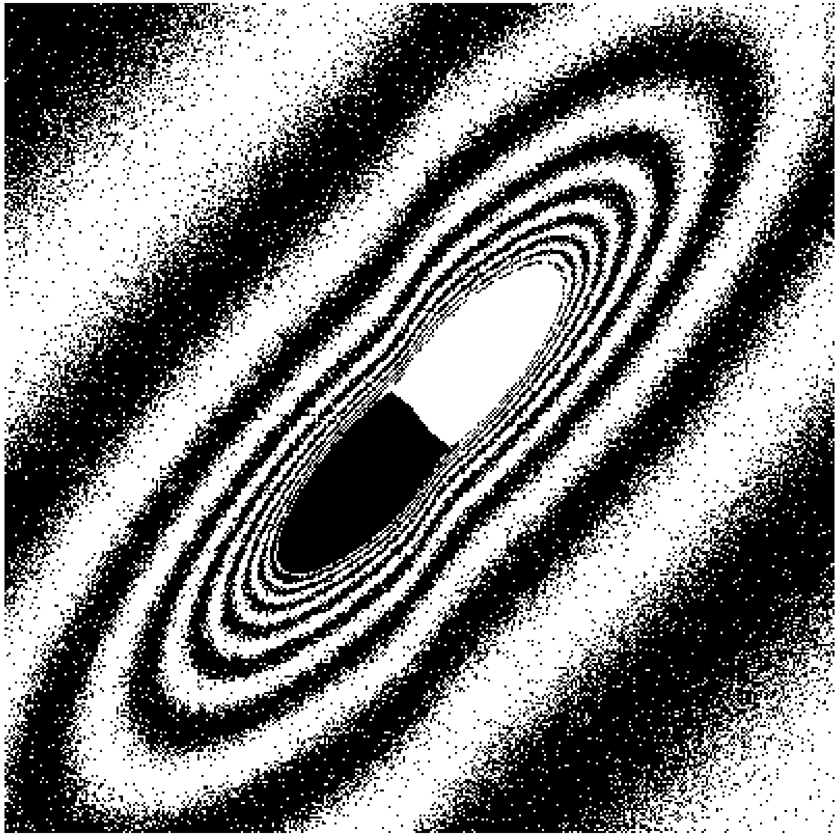
Basin boundary (x-y plane, with $z = 0$)

$\gamma = 1.3$



Basin boundary (x-y plane, with $z = 0$)

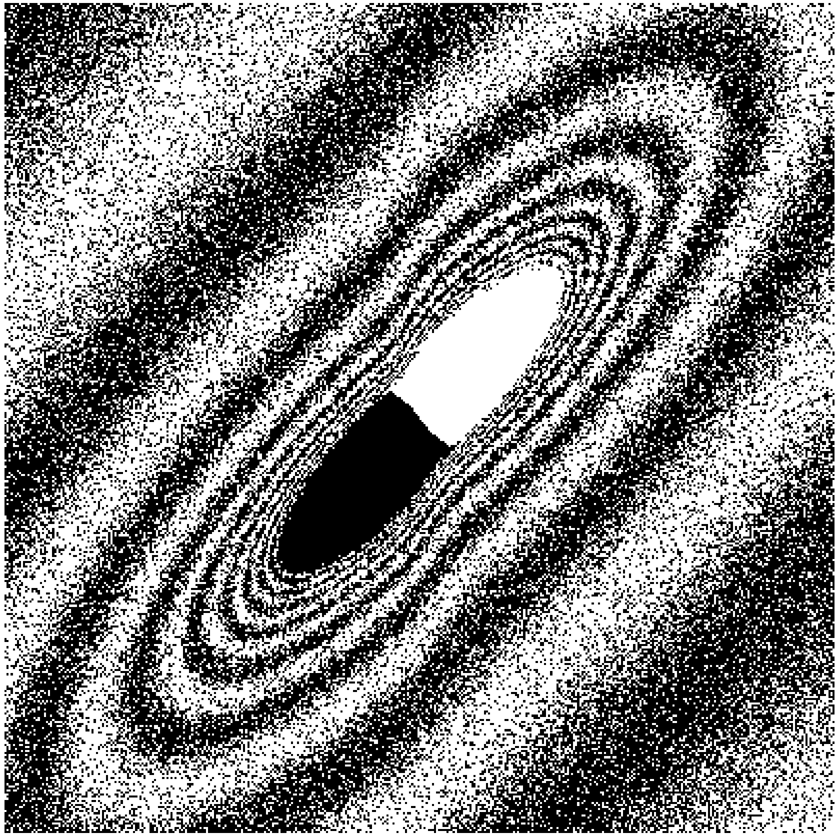
$\gamma = 1.238$



Boundary is suddenly blurred.

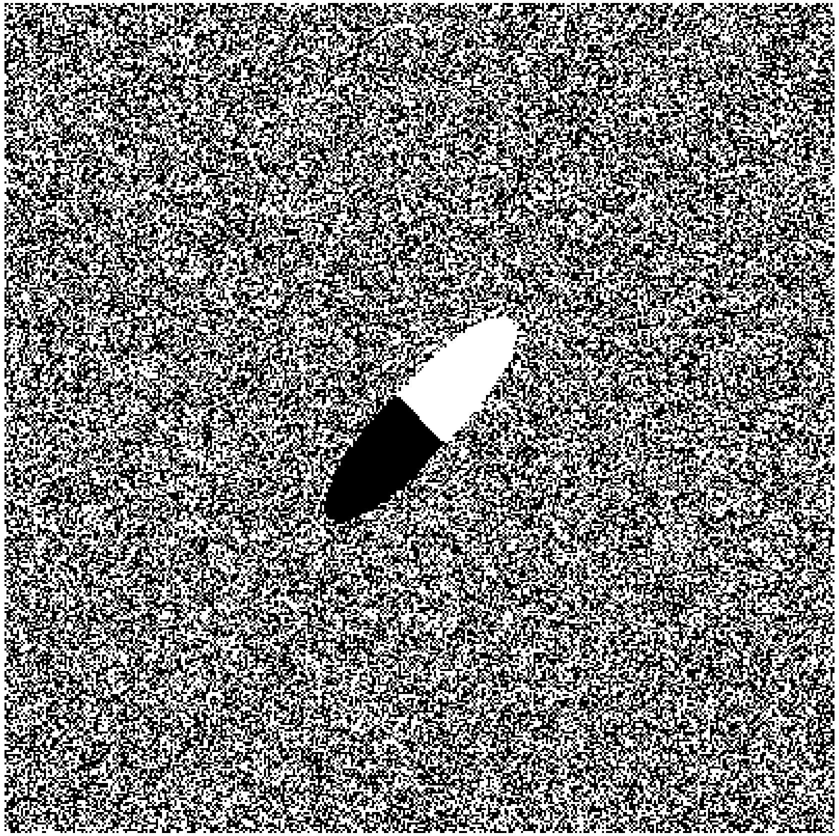
Basin boundary (x-y plane, with $z = 0$)

$\gamma = 1.234$



Basin boundary (x-y plane, with $z = 0$)

$\gamma = 1.2$



not continuously separable.

Fractal basin is not appeared !

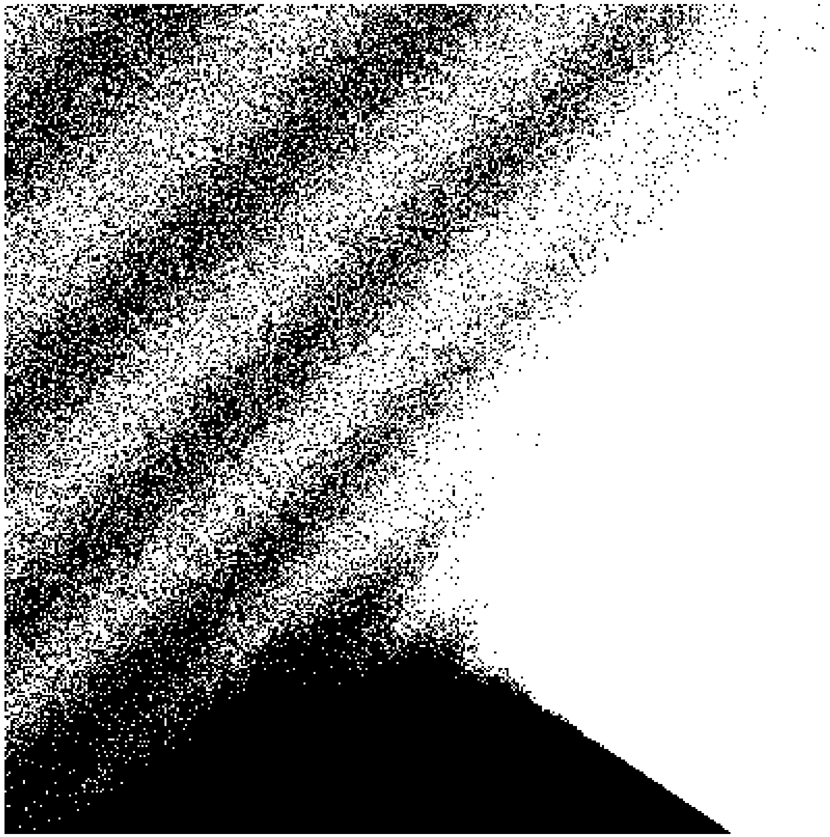
$\gamma = 1.4$

Fractal basin is not appeared !

$\gamma = 1.234$

Fractal basin is not appeared !

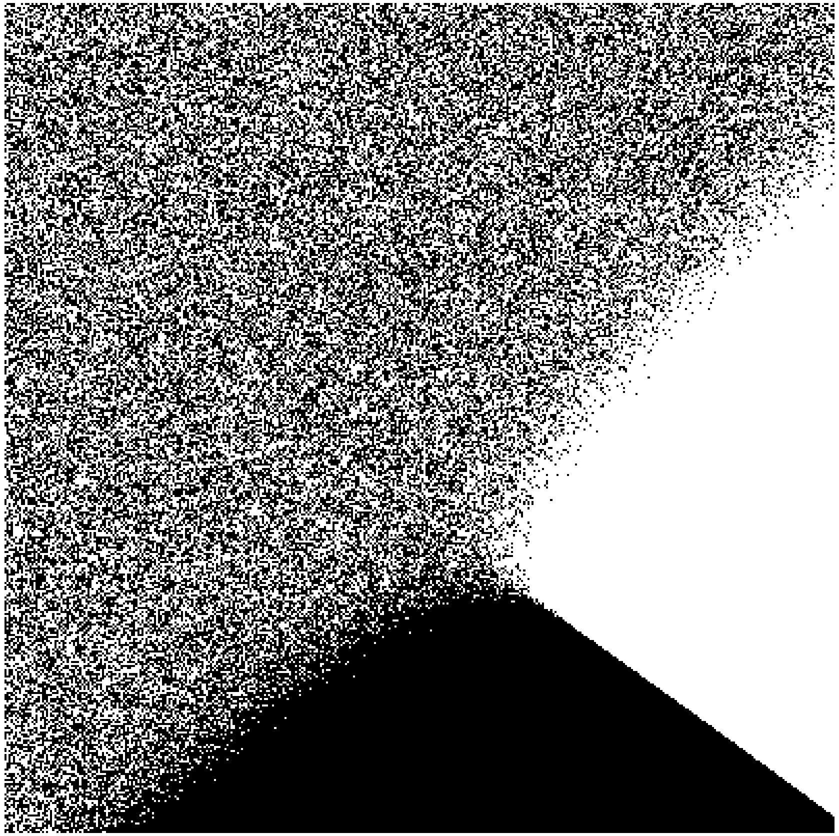
$\gamma = 1.238$



Boundary is suddenly blurred.

Fractal basin is not appeared !

$$\gamma = 1.2$$



God only knows which.

Conclusions

The extended BVP oscillator

- analyses of bifurcations and chaos
- Jack-in-the-box phenomenon
- Blurred basin boundary

Future problems

- investigation of the stable manifold of the origin
- 3D structure of the basin boundary (stable manifold)